NORTH SYDNEY BOYS HIGH SCHOOL

2024 Year 11 Assessment Task 3

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time – 90 minutes
- Write using blue or black pen
- Diagrams may be drawn in pencil
- NSBHS approved calculators may be used
- All necessary working should be shown in every question
- Answer the multiple choice questions on the sheet provided
- Each new question is to be answered in a **SEPARATE BOOKLET**
- Marks may not be awarded for carelessly or badly arranged work.
- Attempt all questions

Class Teacher:

- Ms Lee
- Mr Berry
- Mr Umakanthan
- Dr Vranešević
- Mrs Moss
- Mr Ireland
- Ms Cai/Mr Lott

Student Number:_____

(To be used by the exam markers only.)

Question No	1-5	6	7	8	9	10	Total	Total %
	/5	/12	/12	/12	/12	/12	/65	/100

Question 1

Which of the following is equivalent to $\sin(A - B)$?

(A) $\sin A \cos B(1 - \cot A \tan B)$

(B) $\sin A \cos B(1 + \cot A \tan B)$

(C) $\cos A \sin B\ (1 - \cot A \tan B)$

(D) $\cos A \sin B(1 + \cot A \tan B)$

Question 2

Given the letters ABCDEF. How many arrangements of the letters in a line are there such that it doesn't start with A or B?

(A) 5040

(B) 3600

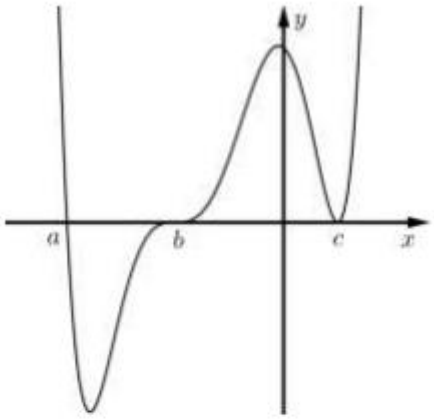
(C) 4320

(D) 3000

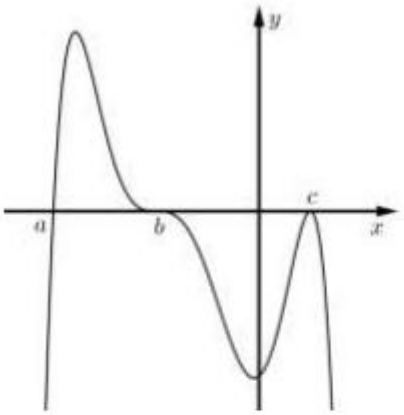
Question 3

A polynomial is defined as $P(x) = (x - a)(x - b)^3(x - c)^2$ if $ab > 0$ and $bc < 0$ which of the following could be the graph of $P(x)$?

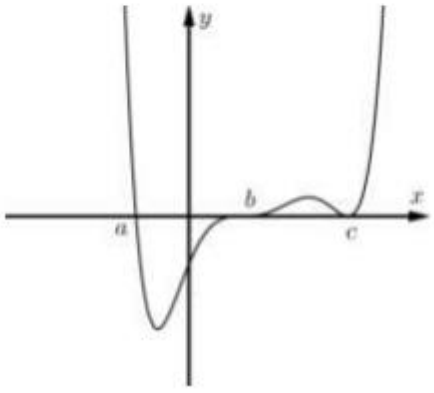
(A)



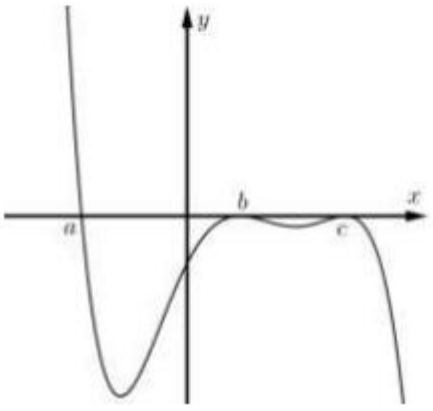
(B)



(C)

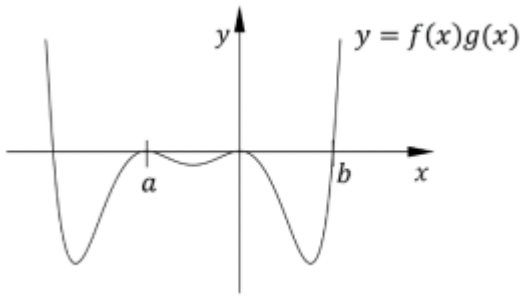


(D)



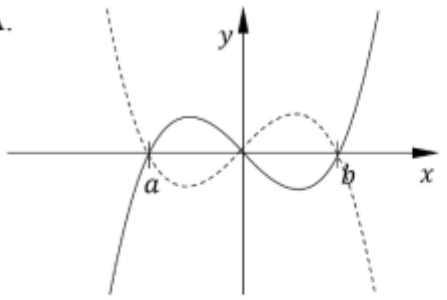
Question 4

The diagram shows the graph of the product of the functions of the functions $f(x)$ and $g(x)$.

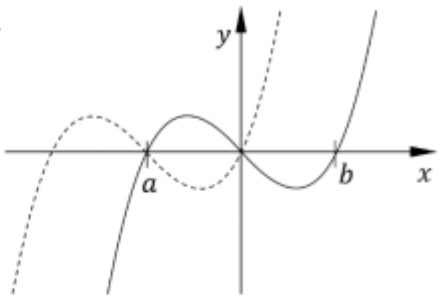


Which of the following best represents the graph of both $f(x)$ and $g(x)$?

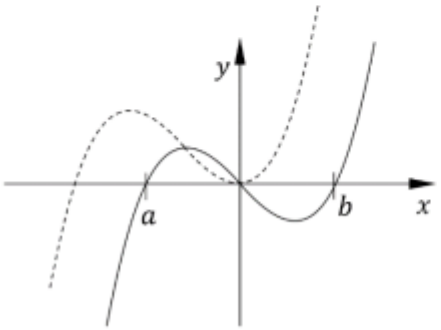
A.



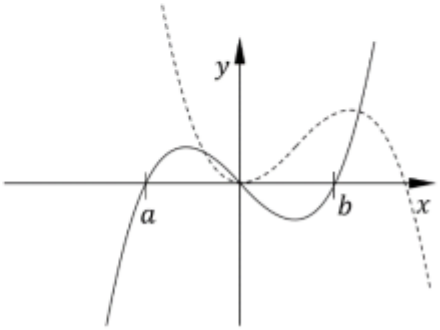
B.



C.



D.



Question 5

For the equation $2x^3 - 4x^2 + 6x - 1 = 0$, with roots α, β and γ, what is the value of

$$\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}?$$

(A) -6

(B) -4

(C) -2

(D) 2

Question 6 (12 Marks)

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3-2x)^6$, giving each term in its simplest form. **2**

(b) A class consists of 10 boys and 12 girls. How many ways are there of selecting a committee of 3 boys and 2 girls from this class? **1**

(c) Solve $|4x-1|+x<4$ **2**

(d) Fully factorise $P(x)=x^4-2x^3-23x^2-12x+36$ **2**

(e) The polynomial $P(x)=8x^4-38x^3+ax+b$ has a double root at $x=3$.

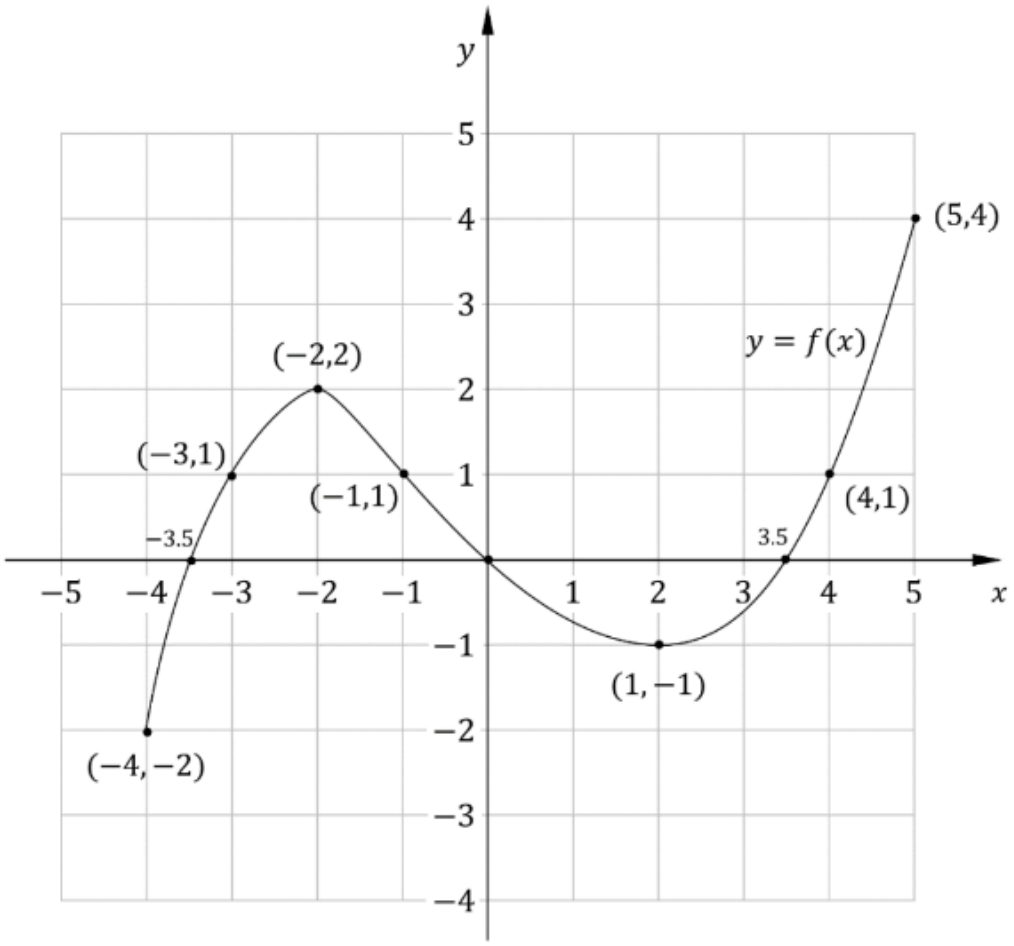
Find the values of a and b, where a and b are real numbers. **3**

(f) Find the Cartesian equation of the curve represented by the parametric equations: **2**

$x=2t-1$
$y=16t^2-22t$

Question 7 (12 Marks)

(a) The graph shows $y = f(x)$, which is defined in the domain $[-4, 5]$.



On the diagrams in your answer booklet for Question 2, sketch the following, showing all relevant features.

(i) $y = f(|x|)$ **2**

(ii) $|y| = f(x)$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **2**

(iv) $y = \sqrt{f(x)}$ **2**

(b) Find the exact value, showing full working, of:

$$\cos\left[2\sin^{-1}\left(\frac{3}{8}\right)\right]$$ **2**

(c) Find the term independent of x in the binomial expansion of:

$$\left(2x - \frac{1}{x^2}\right)^9$$ **2**

Question 8 (12 Marks)

(a) Solve the inequality

$$\frac{1}{x-2}-\frac{1}{x}<0$$ **3**

(b) Consider the function $f(x)=2\cos^{-1}(x-1)$ where $1\leq x\leq 2$.

(i) Sketch $y=f(x)$ clearly showing the coordinates of the endpoints **2**

(ii) Find the equation of the inverse function $y=f^{-1}(x)$ and state its domain. **2**

(c) (i) How many arrangements can be made of the word GATTACA? **1**

(ii) How many of these will contain the word CAT? **2**

(d) If $x=\tan\theta+\sec\theta$, use the t – formula to fully simplify.

$$\frac{x^2-1}{x^2+1}$$

Write your final answer in terms of θ **2**

Question 9 (12 Marks)

(a) (i) Use the binomial theorem to find an expression for

$(1+x)^{2n} + (1-x)^{2n}$ **2**

(ii) Hence evaluate $1 + \binom{20}{2} + \binom{20}{4} + \cdots \binom{20}{20}$ **2**

(b) Solve the equation:

$\sin^{-1} x - \cos^{-1} x = \sin^{-1}(7x - 4)$ **3**

(c) If $(1+kx)^n = 1 - 28x + 336x^2 - \cdots$, find the values of k and n. **3**

(d) Albert, Brandon, and Charlie all submitted an identical assignment for their Investigative Task. The assignment received a mark of 17 out of 20.

Rather than giving them zero for the assignment the teacher said that they could still have the 17 marks but as the assignments were identical, they would have to divide up the 17 marks amongst themselves.

How many ways could this be done? **2**

Question 10 (12 Marks)

(a) (i) Show that:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$ **1**

(ii) Hence find all the solutions to,

$$\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta = 0 \text{ for } 0 \leq \theta \leq 2\pi.$$ **3**

(b) Given that the roots of $x^{2024} + 4x^{20} - 20x^4 + 1 = 0$ are α_1, α_2, α_3 , ... and α_{2024}.
Evaluate $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)\ldots(1+\alpha_{2024})$. **2**

(c) A very boring card game is played as follows:

- The player begins with regular deck of cards face down and turns over the cards one by one counting off their position number.
- If the card turned over matches the position number, the player wins the game and stops playing.
- Otherwise, the player will turn over the next card and continue to play.

For example, if the first card turned over is an Ace the player wins on the first draw and stops playing. If the second card is a 2 (and the first card wasn't an Ace) the player wins on the second draw and stops playing, and so on for the remaining cards.

How many possible games can be played where the player wins on the third card? **3**

(d) Let α, β and γ be the distinct roots of the polynomial $P(x) = x^3 - 22x^2 + 80x - 67$.
It is given that there exist real numbers A, B, and C such that:

$$\frac{1}{x^3 - 22x^2 + 80x - 67} = \frac{A}{x-\alpha} + \frac{B}{x-\beta} + \frac{C}{x-\gamma}$$

for all $x \notin \{\alpha, \beta, \gamma\}$.

Evaluate $\frac{1}{A} + \frac{1}{B} + \frac{1}{C}$. **3**