SHORE

Examination Number:

Set:

## Year 12
## HSC Assessment Task 4 - Trial HSC
## Monday 26 August 2019

# Mathematics Extension 1

**General Instructions**

- Reading time − 5 minutes
- Working time − 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A NESA reference sheet is provided
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- Start each of Questions 11–14 in a new writing booklet
- In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked your examination number and "N/A" on the front cover

**Total marks – 70**

**Section I** Pages 3–6

**10 marks**

- Attempt Questions 1−10
- Allow about 15 minutes for this section

**Section II** Pages 7–11

**60 marks**

- Attempt Questions 11−14
- Allow about 1 hour 45 minutes for this section

**Note: Any time you have remaining should be spent revising your answers.**

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## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the Multiple Choice Answer Sheet for Questions 1–10.

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**1** What is the primitive function of $\frac{1}{2}\sin 2x$?

(A) $\frac{1}{4}\cos 2x + c$

(B) $-\frac{1}{4}\cos 2x + c$

(C) $\cos 2x + c$

(D) $-\cos 2x + c$

**2** If $x = e^y$ then what is the value of $\frac{dy}{dx}$ when $x = \frac{1}{e}$?

(A) $e$

(B) $\frac{1}{e}$

(C) 1

(D) 0

**3** What is the value of $\int_0^3 \frac{dx}{9 + x^2}$ ?

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

**4** What is the size of the acute angle, to the nearest degree, between the straight lines $4x - 5y + 1 = 0$ and $2x + 7y - 4 = 0$?

(A) 22°

(B) 23°

(C) 54°

(D) 55°

**5** What is the domain and range of $y = 2\sin^{-1}\frac{x}{3}$?

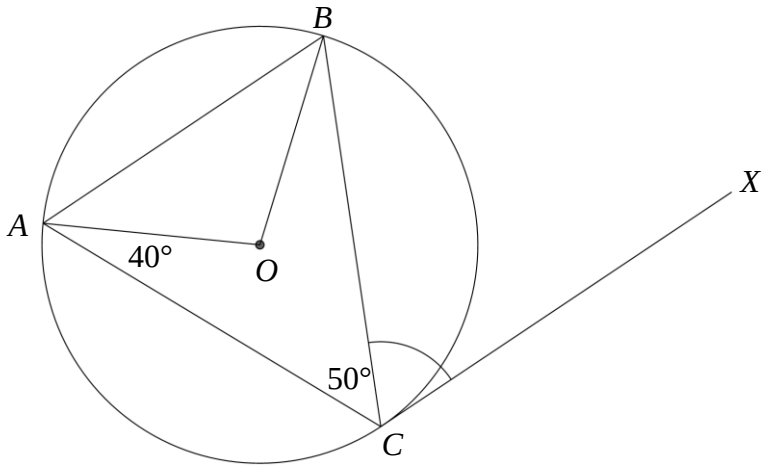
(A) Domain: $-\frac{1}{3} \le x \le \frac{1}{3}$ and range: $-\pi \le y \le \pi$

(B) Domain: $-\frac{1}{3} \le x \le \frac{1}{3}$ and range: $-3\pi \le y \le 3\pi$

(C) Domain: $-3 \le x \le 3$ and range: $-\pi \le y \le \pi$

(D) Domain: $-3 \le x \le 3$ and range:. $-3\pi \le y \le 3\pi$

**6** In the diagram below *A*, *B* and *C* are points on the circumference of circle centre *O*, while tangent *XC* meets the circle at *C*. As shown, $\angle ACB = 50°$ and $\angle OAC = 40°$.




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What is the size of $\angle BCX$ ?

(A) 30°

(B) 40°

(C) 70°

(D) 80°

**7** The number $N(t)$ of individuals in a population is given by $N(t) = 100 + 200e^{-0.1t}$ after $t$ years. How many years does it take for the population size to fall to half of the initial population?

(A) $10 \log_e 2$ years

(B) $10 \log_e 3$ years

(C) $10 \log_e 4$ years

(D) $10 \log_e 6$ years

**8** Let $\alpha, \beta$ and $\gamma$ be the roots of $y^3 - 5y + 1 = 0$. What is the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$?

(A) $-5$

(B) $-1$

(C) 1

(D) 5

**9** Which of the following is the general solution of $2\cos\frac{2x}{3} = 1$ ?

(A) $3n\pi \pm \frac{\pi}{2}$

(B) $3n\pi \pm \frac{\pi}{3}$

(C) $n\pi \pm \frac{\pi}{2}$

(D) $n\pi \pm \frac{\pi}{3}$

**10** A particle is undergoing simple harmonic motion about a fixed point *O* on a line. At time *t* seconds it has displacement *x* metres from *O* given by $x = a\cos nt$ where $a > 0$ and $n > 0$. The period of the motion is *T* seconds. What is the time taken by the particle to move from its starting position to a point half-way towards *O* ?

(A) $\frac{T}{12}$

(B) $\frac{T}{9}$

(C) $\frac{T}{8}$

(D) $\frac{T}{6}$

**End of Section I**

## Section II

**60 marks**
**Attempt Questions 11–14**
**Allow about 1 hour 45 minutes for this section**
Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use a SEPARATE writing booklet

(a) Evaluate $m$ if the remainder of 39 is obtained when $3x^4 - mx^2 - 1$ is divided by $x - 2$. **1**

(b) Point $A$ has the coordinates $(-2, 3)$ and point $B$ has the coordinates $(3, -4)$. Find the coordinates of the point, $P$, that divides $AB$ internally in the ratio 3:2. **2**

(c) Solve $\dfrac{x^2 - 12}{x} \leq 1$. **3**

(d) A function is defined as $f(x) = \dfrac{5x + 1}{x^2 + x - 2} - \dfrac{3}{x + 2}$.

(i) Show that $f(x) = \dfrac{2}{x - 1}$ where $x \neq -2$ and $x \neq 1$. **1**

(ii) Find $f^{-1}(x)$. **1**

(e) The function $f(x) = \log_e 2x - \cos x$ has a zero near $x = 0.8$. **3**

Use one application of Newton's Method to obtain another approximation to this zero. Give your answer correct to 2 significant figures.

(f) (i) Simplify $\dfrac{\sin 2x}{1 + \cos 2x}$. **2**

(ii) Hence, or otherwise, express the exact value of $\tan 15^\circ$ as a fraction with a rational denominator. **2**

**End of Question 11**

**Question 12** (15 marks) Use a SEPARATE writing booklet

(a) $P(6p, 3p^2)$ and $Q(6q, 3q^2)$ are two points on the parabola $x^2 = 12y$ such that the chord $PQ$ is parallel to the line $y = 2x$. $M$ is the midpoint of $PQ$.

(i) Show that $p + q = 4$. **2**

(ii) Show that $M$ lies on a straight line and state the equation of the line. **2**

(b) Roberto is 120 metres east of a building, $h$ metres high, and he can see the top of the building at an angle of elevation of 30°. Becky is 202 metres south of this building and she can see the top of this building at an angle of elevation, $\alpha$. Construct a diagram to show this information and then find the angle of elevation, $\alpha$, correct to the nearest degree. **2**

(c) Prove by Mathematical Induction that $7^n - 1$ is divisible by 6 for $n \geq 1$. **3**

(d) Evaluate $\int_0^2 4x\sqrt{1-\frac{x}{2}}\,dx$ using the substitution $u = 1 - \frac{x}{2}$. **3**

(e) A particle is moving in simple harmonic motion about the origin, $O$, such that its velocity $v\ ms^{-1}$ satisfies $v^2 = 9\left(4 - x^2\right)$, where $x$ is the displacement of the particle from $O$. The initial velocity of the particle is zero. How many seconds will it take the particle to first reach $O$? **3**

**End of Question 12**

**Question 13** (15 marks) Use a SEPARATE writing booklet

(a) Find the coefficient of $x^5$ in the expansion of $\left(\frac{1}{x^3} + 2x^2\right)^{15}$. **2**

(b) The volume, $V$, of a sphere of radius, $r$ mm, is increasing at a constant rate of 160 $\text{mm}^3$ per second. Note that $V = \frac{4}{3}\pi r^3$ and $\text{Surface Area} = 4\pi r^2$. **3**

Find the rate of change of the surface area, $A$, of the sphere when the radius is 40 mm.

(c) Let $h(x) = \tan^{-1}\sqrt{x^2 - 1} + \sin^{-1}\frac{1}{x}$ where $x \geq 1$. Show that $h(x)$ is constant. **3**

(d) The acceleration of a particle moving along the $x$ axis is given by $\ddot{x} = 1 + \frac{x}{x^2 + 1}$, where $x$ is displacement from the origin, $O$, after t seconds, and $v$ is velocity. Find $v^2$ in terms of $x$ given that $v = 2$ when $x = 2$. **3**

(e) (i) Write $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and $0 \leq \alpha \leq \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve the equation $\cos x - \sqrt{3}\sin x = 1$ for $0 \leq x \leq 2\pi$. **2**

**End of Question 13**

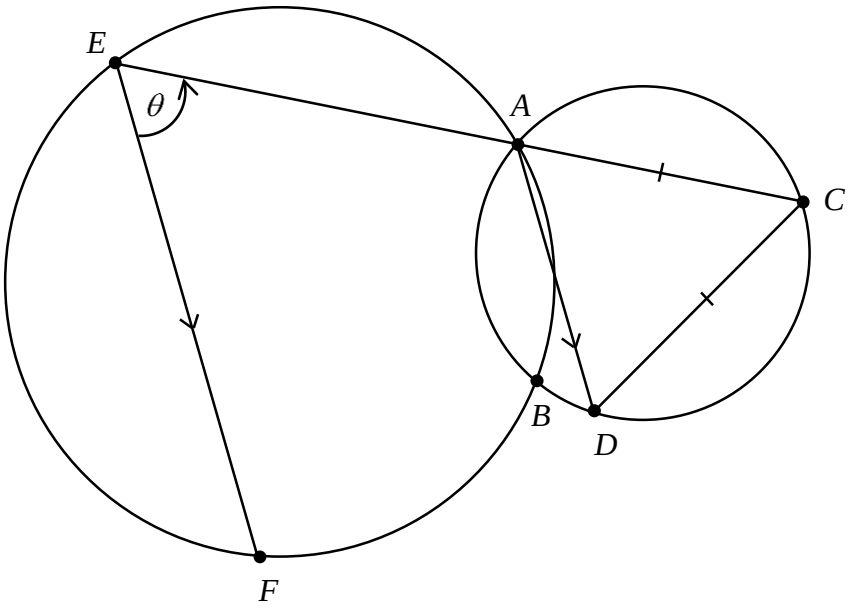
**Question 14** (15 marks) Use a SEPARATE writing booklet

(a) (i) Expand $(1-x)^n$ using the binomial theorem. **1**

(ii) Show that $\int_0^1 (1-x)^n\,dx = {}^nC_0 - \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 - \ldots + \frac{(-1)^n}{n+1}\,{}^nC_n$. **2**

(iii) Show that $\frac{1}{n+1} = \sum_{r=0}^{n} (-1)^r \frac{1}{r+1}\,{}^nC_r$. **1**

(b) In the diagram, two circles intersect at the points *A* and *B*. **3**
*EF* is a chord in the large circle. The line *EA* produced meets the smaller circle at *C*. The line from *A*, parallel to *EF*, intersects the smaller circle at *D* such that *AC* = *CD*. Let $\angle FEA = \theta$.

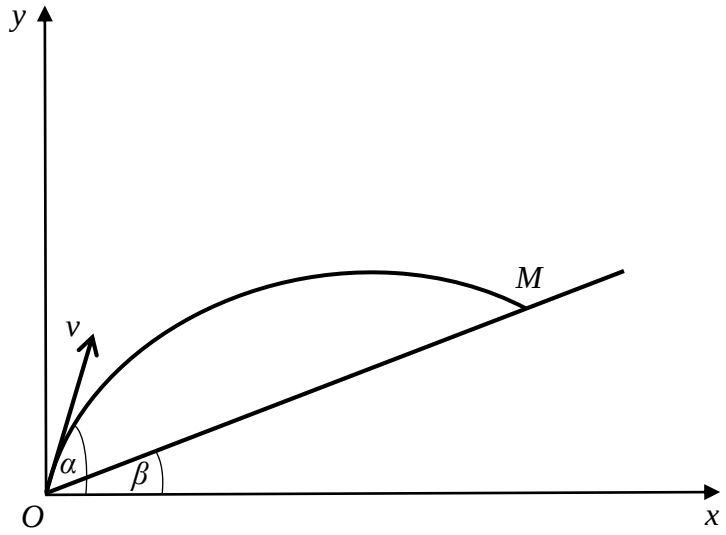



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Copy the diagram into your booklet and prove that *F*, *B* and *C* are collinear points.

**Question 14 continues on the next page**

(c) A ball is thrown from $O$ with initial velocity $v$ and angle $\alpha^o$ to the horizontal. A fixed straight line passes through $O$ and is on an incline to the horizontal at angle $\beta^o$, where $\alpha > \beta$. The ball hits the line at $M$. Let the length $OM = d$ metres and the acceleration due to gravity is $g\,\text{ms}^{-2}$.




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(i) Given that $x = vt\cos\alpha$ and $y = -\frac{1}{2}gt^2 + vt\sin\alpha$, show that the equation of path of the ball is $y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2v^2}$. **2**

(ii) Write down the coordinates of $M$, in terms of $d$ and $\beta$. **1**

(iii) Hence show that $d = \frac{2v^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$. **2**

(iv) Show that the maximum value of $d$ occurs when $\alpha = \frac{1}{2}\left(\beta + \frac{\pi}{2}\right)$. **3**

Do not prove that this is a maximum.

**END OF PAPER**

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