MEREWETHER
HIGH SCHOOL

2018 HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 4

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided at the back of this paper.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- Submit ALL work at the end of the examination.
- Diagrams are NOT drawn to scale.

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

Section II – 60 marks (pages 6 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 The equation $2x^3 + x^2 - 13x + 6 = 0$ has roots α, $\frac{1}{\alpha}$ *and* β. What is the value of β?

A. 3

B. –3

C. 2

D. –6

2 Which of the following is a simplification of $4\log_e \sqrt{e^x}$?

A. $4\sqrt{x}$

B. $\frac{1}{2}x$

C. $2x$

D. x^2

3 Which of the following is an expression for $\frac{dy}{dx}$ if $x = \frac{1}{2}at$ and $y = at^2$?

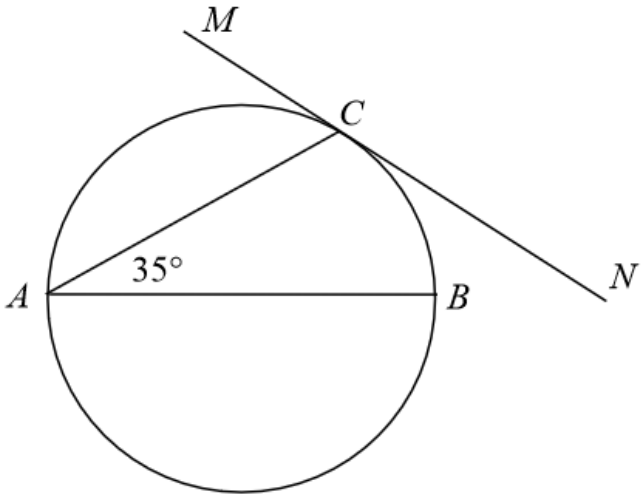
A. t

B. $2t$

C. $2at$

D. $4t$

4 In the triangle, AB is the diameter of the circle and MCN is the tangent to the circle at C. $\angle CAB = 35°$. What is the size of $\angle MCA$?



NOT TO SCALE

A. 35°

B. 45°

C. 55°

D. 65°

5 Which of the following is an expression for $\int \sin^2 2x dx$?

A. $\frac{1}{2}x - \frac{1}{8}\sin 4x + C$

B. $\frac{1}{2}x - \frac{1}{4}\sin 4x + C$

C. $\frac{1}{2}x + \frac{1}{8}\sin 4x + C$

D. $\frac{1}{2}x + \frac{1}{4}\sin 4x + C$

6 What is the value of $\lim\limits_{x\to 0}\left(\dfrac{\sin\frac{1}{3}x}{2x}\right)$?

A. $\dfrac{1}{6}$

B. $\dfrac{2}{3}$

C. $\dfrac{3}{2}$

D. 6

7 Which of the following is the range of the function $y = 2\sin^{-1}x + \dfrac{\pi}{2}$?

A. $-\pi \le y \le \pi$

B. $-\dfrac{\pi}{2} \le y \le \dfrac{3\pi}{2}$

C. $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

D. $-\pi \le y \le \dfrac{3\pi}{2}$

8 Which of the following is an expression for $\dfrac{d}{dx}\sin^{-1}(2x-1)$?

A. $\dfrac{-1}{\sqrt{x(x-1)}}$

B. $\dfrac{-1}{2\sqrt{x(x-1)}}$

C. $\dfrac{1}{2\sqrt{x(x-1)}}$

D. $\dfrac{1}{\sqrt{x(1-x)}}$

9 If $\frac{dN}{dt} = 0.1\left(N - 100\right)$ and $N = 300$ when $t = 0$, which of the following is an expression for N?

A. $200 + 100e^{0.1t}$

B. $300 + 100e^{0.1t}$

C. $100 + 200e^{0.1t}$

D. $100 + 300e^{0.1t}$

10 A particle is performing simple harmonic motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line where x is given by $x = 4\sin^2 t - 1$. Where is the centre of motion?

A. $x = -1$

B. $x = 0$

C. $x = 1$

D. $x = 2$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\frac{4x-1}{x+2} \geq 1$ **2**

(b) $A(-3,1)$ and $B(1,-2)$ are two points. Find the coordinates of point P that divides the interval AB externally in the ratio $3:1$. **2**

(c) Find the number of ways in which the letters of the word CIRCLE can be arranged in a row so that the two vowels are in the two end positions. **2**

(d) $P\left(2at, at^2\right)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0,a)$

(i) Show that the normal to the parabola at P has gradient $-\frac{1}{t}$. **1**

(ii) If θ is the acute angle between the normal to the parabola and the line PF show that $\tan\theta = |t|$. **2**

Question 11 continues on page 7

(e) The equation $x^3 - 6x^2 + 4x + 2 = 0$ has roots α, β and γ. Find the value of

(i) $\dfrac{\alpha}{2} + \dfrac{\beta}{2} + \dfrac{\gamma}{2}$ **1**

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma}$ **2**

(f) Use the substitution $u = x + 2$ to find $\displaystyle\int \frac{3x+5}{\sqrt{x+2}}\,dx$. **3**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\frac{d}{dx}\left(e^x \tan^{-1} x\right)$. **2**

(b) Use the substitution $t = \tan\frac{x}{2}$ to show that $\frac{1+\cos x+\sin x}{1-\cos x+\sin x} = \cot\frac{x}{2}$. **3**

(c) Use the method of Mathematical Induction to show that $5^n - 4n - 1$ is divisible by 4 for all positive integers $n \geq 2$. **3**

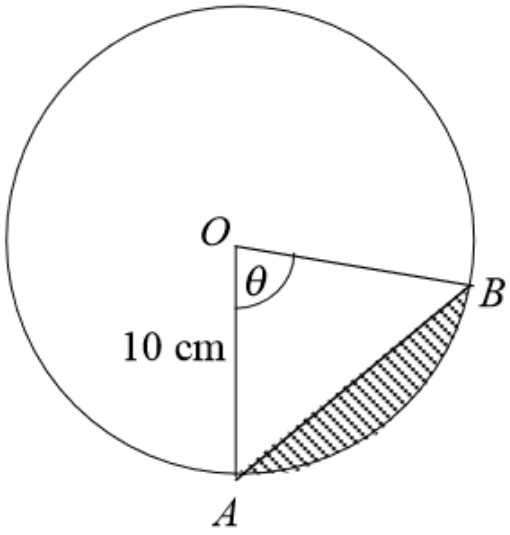
(d) Consider the function $f(x) = 2\cos^{-1}(x-1)$ where $1 \leq x \leq 2$.

(i) Sketch the curve $y = f(x)$, showing clearly the coordinates of the endpoints. **2**

(ii) Find the equation of the inverse function $f^{-1}(x)$ and state its domain. **2**

Question 12 continues on page 9

(e)



A circle has centre O and radius 10 cm. OA is a fixed radius of the circle. OB is a variable radius which moves so that $\angle AOB = \theta$ is increasing at a constant rate of 0.01 radians per second. The minor segment of the circle cut off by the chord AB has shaded area S cm^2.

Find the rate at which S is increasing when $\theta = \dfrac{\pi}{3}$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

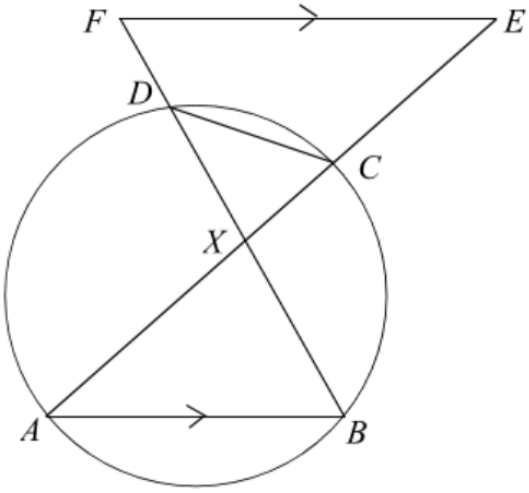
(a) Consider the equation $x^3 + 2x - 7 = 0$

(i) Show that the equation has a root α such that $1 < \alpha < 2$. **2**

(ii) Show that α is the only real root of the equation. **1**

(iii) Use one application of Newton's Method with an initial approximation $\alpha = 1.5$ to find the next approximation for α correct to 1 decimal place. **2**

(b) **3**

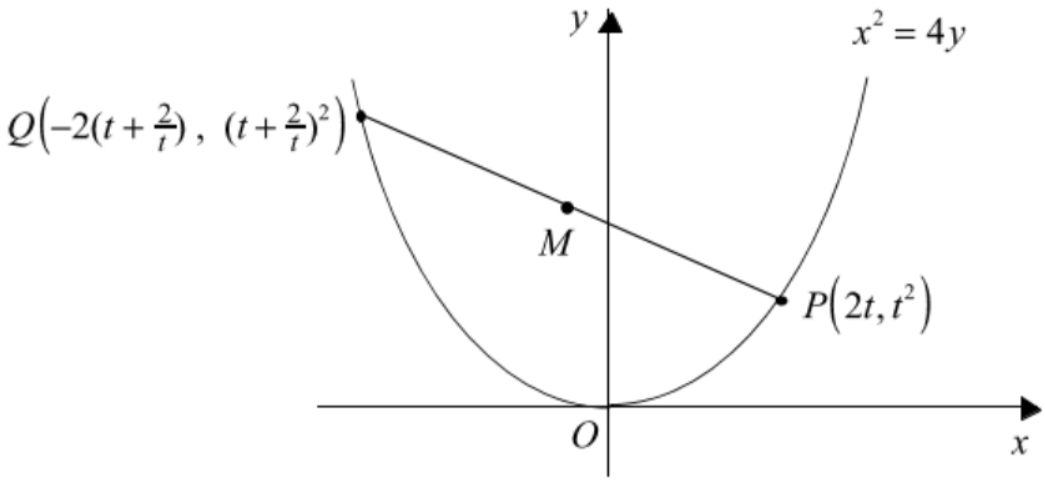


AC and BD are two chords of a circle which intersect at point X inside the circle. E is a point on AC produced and F is a point of BD produced such that $FE \| AB$. Show that $DCEF$ is a cyclic quadrilateral.

(c) Find the exact value of $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx$. **3**

Question 13 continues on page 11

(d)



$P\ (2t, t^2)$ and $Q\ (-2(t+\frac{2}{t}), (t+\frac{2}{t})^2)$ are two points on the parabola $x^2 = 4y$.
M is the midpoint of PQ

(i) By considering the gradient of QP, show that QP is normal to the parabola at P. **2**

(ii) Find the equation of the locus at M as P moves on the parabola. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{3}$, show that $\sin^{-1} x = \frac{5\pi}{12}$. **2**

(b) A particle moves along the x – axis according to the equation $x = \sin 2t - \sqrt{3}\cos 2t$, where x is in metres and t is in seconds.

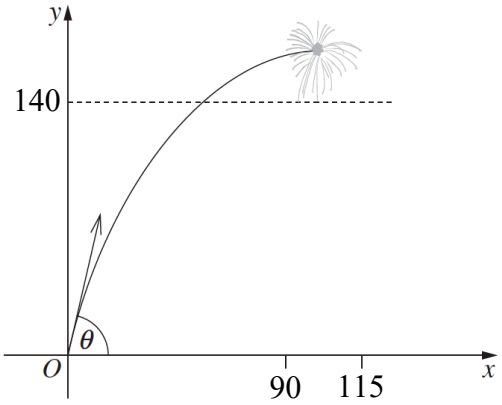
(i) Express x in the form $R\sin(2t-\alpha)$, where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Show that the particle is moving in simple harmonic motion. **2**

(iii) State the period of the particle's motion. **1**

(iv) When is the first time that the particle is 2 metres right of the origin? **2**

(c) A firework is fired from O, on level ground, with velocity 60 metres per second at an inclination θ. The equations of motion of the firework are $x = 60t\cos\theta$ and $y = 60t\sin\theta - 5t^2$ (Do NOT prove this). The firework explodes when it reaches its maximum height.



NOT TO SCALE

(i) Show that the firework explodes at a height of $180\sin^2\theta$ metres. **2**

(ii) Show that the firework explodes at a horizontal distance of $180\sin 2\theta$. **1**

(iii) For best viewing, the firework must explode at a horizontal distance between 90 metres and 115 metres from O, and at least 140 metres above the ground. For what values of θ will this occur? **3**

End of paper

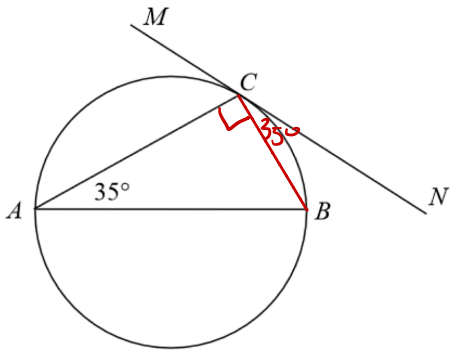
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Year 12 Extension 1 Mathematics Trial HSC Solutions 2018

Question	Suggested solution	Marking guidelines	Markers Notes
1.	$\alpha \times \frac{1}{\alpha} \times \beta = -\frac{d}{a}$ $\beta = \frac{-6}{2}$ $= -3$	B	
2.	$4 \log_e \sqrt{e^x} = 4 \log_e e^{\frac{x}{2}}$ $= 4\left(\frac{x}{2}\right) \log_e e$ $= 2x$	C	
3.	$x = \frac{1}{2}at$ $\quad y = at^2$ $\frac{dx}{dt} = \frac{a}{2}$ $\quad \frac{dy}{dt} = 2at$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ $= 2at \times \frac{2}{a}$ $= 4t$	D	Don't sub $a = \frac{2x}{t}$ into y for $y = 2xt$ to find $\frac{dy}{dx}$. This is not differentiable as 3 variables are involved.

Question	Suggested solution	Marking guidelines	Markers Notes

4.



NOT TO SCALE

$\angle ACB = 90°$ ($\angle$ in semi circle)

$\angle NCB = 35°$ ($\angle$ in alternate segment)

$\therefore MCA = 180 - (90+35)$ ($\angle$'s on straight line)

$= 55°$

C

5. $\int \sin^2 2x\,dx = \int \frac{1}{2}(1-\cos 4x)\,dx$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 4x\right)dx$

$= \frac{1}{2}x - \frac{1}{8}\sin 4x + C$

A

6. $\lim_{x\to 0} \frac{\sin \frac{1}{3}x}{2x} = \frac{1}{2}\lim_{x\to 0} \frac{\sin \frac{1}{3}x}{x}$

$= \frac{(\frac{1}{3})}{2}\lim_{x\to 0} \frac{\sin \frac{1}{3}x}{\frac{1}{3}x}$

$= \frac{1}{6} \times 1$

$= \frac{1}{6}$

A

Question	Suggested solution	Marking guidelines	Markers Notes

7. $y = 2\sin^{-1}x + \frac{\pi}{2}$

range of $y = \sin^{-1}x$: $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$-\pi \le 2\sin^{-1}x \le \pi$

$-\frac{\pi}{2} \le 2\sin^{-1}x + \frac{\pi}{2} \le \frac{3\pi}{2}$

$\therefore$ range of $y = 2\sin^{-1}x + \frac{\pi}{2}$: $-\frac{\pi}{2} \le y \le \frac{3\pi}{2}$ B

8. $\frac{d}{dx}\sin^{-1}(2x-1) = \frac{1}{\sqrt{1-(2x-1)^2}} \times 2$

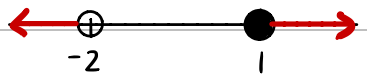
$= \frac{2}{\sqrt{4x-4x^2}}$

$= \frac{2}{\sqrt{4x(1-x)}}$

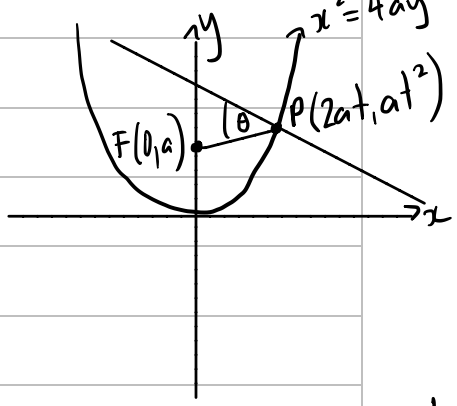
$= \frac{1\cancel{2}}{1\cancel{2}\sqrt{x(1-x)}}$

$= \frac{1}{\sqrt{x(1-x)}}$ D

Question	Suggested solution	Marking guidelines	Markers Notes
9	$\frac{dN}{dt} = 0.1(N-100)$		
	$\therefore \quad N = 100 + Ae^{0.1t}$ for some constant A		
	When $N=300$ and $t=0$		
	$300 = 100 + Ae^{0.1(0)}$		
	$200 = A$		
	$\therefore \quad N = 100 + 200e^{0.1t}$	C	
10.	$x = 4\sin^2 t - 1$		
	$= 2(1-\cos 2t) - 1$		
	$= 1 - 2\cos 2t$		
	$\therefore$ centre is at $x = 1$	C	
	B C D C A A B D C C		

Question	Suggested solution	Marking guidelines	Markers Notes
11.(a)	$\frac{4x-1}{x+2} \geq 1$		
	$x+2 \neq 0$		
	$x \neq -2$		
	Solve $\frac{4x-1}{x+2} = 1$		
	$4x-1 = x+2$		
	$3x = 3$		
	$x = 1$	1	
			
	Test $x=-3$ $\frac{4(-3)-1}{-3+2} = 13 \geq 1$ ✓		Must show substitution for test
	$x=0$ $\frac{4(0)-1}{0+2} = -\frac{1}{2} \leq 1$ ×		
	$x=2$ $\frac{4(2)-1}{2+2} = \frac{7}{4} \geq 1$ ✓		
	$\therefore x < -2, x \geq 1.$	1	
(b)	$A(-3, 1)$ $B(1, -2)$ externally $3:-1$ ($x_1 y_1$, $x_2 y_2$, $m\ n$)		
	$x = \frac{mx_2 + nx_1}{m+n}$ $y = \frac{my_2 + ny_1}{m+n}$		
	$= \frac{3(1)-1(-3)}{3-1}$ $= \frac{3(-2)-1(1)}{3-1}$		
	$= \frac{6}{2}$ $= \frac{-7}{2}$ ①		
	$= 3$ ①		
	$\therefore P\left(3, -\frac{7}{2}\right)$		

Question	Suggested solution	Marking guidelines	Markers Notes
11(c)	6 letters, 2 c's 4 consonants 2 vowels V C C C C V	1	
	∴ Total arrangements $= \frac{2! \times 4!}{2!}$ $= 4!$ $= 24$	1	* Forgetting to divide by 2! for the 2 C's.
(d) i)	$x^2 = 4ay$ $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{2x}{4a}$ $= \frac{x}{2a}$		
	∴ gradient of tangent at P, $\frac{dy}{dx} = \frac{2at}{2a}$ $= t$		
	∴ gradient of normal $= -\frac{1}{t}$	1	must use differentiation for show that
ii)	$m_{PF} = \frac{at^2 - a}{2at - 0}$ $= \frac{a(t^2-1)}{2at}$ $= \frac{t^2-1}{2t}$	1	



Question	Suggested solution	Marking guidelines	Markers Notes
	$\tan\theta = \left\lvert \frac{\frac{t^2-1}{2t} - \left(-\frac{1}{t}\right)}{1+\frac{1}{2t}(t^2-1)\left(-\frac{1}{t}\right)} \right\rvert$		* struggled with algebra required
	$\left\lvert \frac{\frac{1}{2t}\left[(t^2-1)-(-2)\right]}{1+\frac{1}{2t}(t^2-1)\left(-\frac{1}{t}\right)} \right\rvert$		
	$= \left\lvert \frac{\frac{1}{2t}(t^2+1)}{1-\frac{1}{2t^2}(t^2-1)} \right\rvert$		
	$= \left\lvert \frac{\frac{1}{2t}(t^2+1)}{\frac{1}{2t}(2t^2-1(t^2-1)} \right\rvert$		
	$= \left\lvert \frac{t(t^2+1)}{2t^2-t^2+1} \right\rvert$		
	$= \left\lvert \frac{t(t^2+1)}{t^2+1} \right\rvert$		
	$= \lvert t \rvert$	1	
(e)	$x^3 - 6x^2 + 4x + 2 = 0$		
	(i) $\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \frac{\alpha+\beta+\gamma}{2}$		
	$= \frac{\left(-\frac{b}{a}\right)}{2}$		
	$= \frac{-(-6)}{2}$		
	$= 3$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
(ii)	$\frac{2}{\alpha}+\frac{2}{\beta}+\frac{2}{\gamma}=\frac{2(\beta\gamma+\alpha\gamma+\alpha\beta)}{\alpha\beta\gamma}$	1	
	$=\frac{2\left(\frac{c}{a}\right)}{\left(-\frac{d}{a}\right)}$		
	$=\frac{2(4)}{(-2)}$		
	$=-4$	1	
f)	$\int\frac{3x+5}{\sqrt{x+2}}=\int\frac{3(u-2)+5}{\sqrt{u}}\,du$		
	$u=x+2$ $\quad=\int u^{-\frac{1}{2}}(3u-1)\,du$		$\frac{1}{\sqrt{u}}=u^{-\frac{1}{2}}$ not $u^{\frac{1}{2}}$
	$x=u-2$		
	$\frac{du}{dx}=1$ $\quad=\int 3u^{\frac{1}{2}}-u^{-\frac{1}{2}}\,du$	1	* these are not like terms $\therefore \neq 2u^{-\frac{1}{2}}$
	$\therefore du=dx$ $\quad=\frac{3u^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}-\frac{u^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+C$	1	
	$=2\sqrt{u^3}-2\sqrt{u}+C$		
	$=2\sqrt{(x+2)^3}-2\sqrt{x+2}+C$	1	* Answer must be in same form as question. i.e surds not fractional indices and in terms of x * Mark still awarded if left as fractional indice

Question	Suggested solution	Marking guidelines	Markers Notes

12.(a) $u = e^x \quad v = \tan^{-1}x$

$u' = e^x \quad v' = \dfrac{1}{1+x^2}$ 1

$\dfrac{d}{dx}(e^x \cdot \tan^{-1}x) = \dfrac{e^x}{1+x^2} + \tan^{-1}x \cdot e^x$ 1

$= e^x\left(\dfrac{1}{1+x^2} + \tan^{-1}x\right)$

(b) $\text{LHS} = \dfrac{1+\cos x+\sin x}{1-\cos x+\sin x}$

$= \left(1 + \dfrac{1-t^2}{1+t^2} + \dfrac{2t}{1+t^2}\right) \div \left(1 - \dfrac{(1-t^2)}{1+t^2} + \dfrac{2t}{1+t^2}\right)$ 1

$= \left(\dfrac{1+t^2+1-t^2+2t}{1+t^2}\right) \div \left(\dfrac{1+t^2-1+t^2+2t}{1+t^2}\right)$

$= \left(\dfrac{2+2t}{1+t^2}\right) \div \left(\dfrac{2t^2+2t}{1+t^2}\right)$

$= \dfrac{2(1+t)}{1+t^2} \times \dfrac{1+t^2}{2t(t+1)}$ 1

$= \dfrac{1}{t}$

$= \dfrac{1}{\tan\frac{x}{2}}$

$= \cot\dfrac{x}{2}$ 1

$= \text{RHS}$

Question	Suggested solution	Marking guidelines	Markers Notes
(c)	$5^n - 4n - 1$ divisible by 4 for $n \geq 2$		
	Step 1: Prove true for $n = 2$		
	$5^2 - 4(2) - 1 = 25 - 8 - 1$		
	$= 16$ ($\div$ by 4)		
	$\therefore$ True for $n = 2$		
	Step 2: Assume true for $n = k$		
	i.e $5^k - 4k - 1 = 4M$ where M is a positive integer	1.	
	$5^k = 4M + 4k + 1$		
	Step 3: Prove true for $n = k+1$		
	i.e $5^{k+1} - 4(k+1) - 1$ is divisible by 4		
	$5^{k+1} - 4(k+1) - 1 = 5.5^k - 4k - 4 - 1$		
	$= 5(4M + 4k + 1) - 4k - 5$ (from step 2)		
	$= 20M + 20k + 5 - 4k - 5$		
	$= 20M + 16k$		
	$= 4(5M + 4k)$ ($\div$ by 4)	1	
	Step 4: Conclusion		
	True for $n = k+1$ if true for $n = k$		
	True for $n = 2$		
	$\therefore$ by induction, true for all integers $n \geq 2$	1	

Question	Suggested solution	Marking guidelines	Markers Notes

(d) (i) For $f(x) = 2\cos^{-1}(x-1)$, $1 \le x \le 2$.

when $x = 1$ $f(1) = 2\cos^{-1}(1-1)$

$= 2\cos^{-1} 0$

$= 2\left(\frac{\pi}{2}\right)$

$= \pi$

Domain $\cos^{-1} x$ $-1 \le x \le 1$

$-1 \le x-1 \le 1$

$0 \le x \le 2$

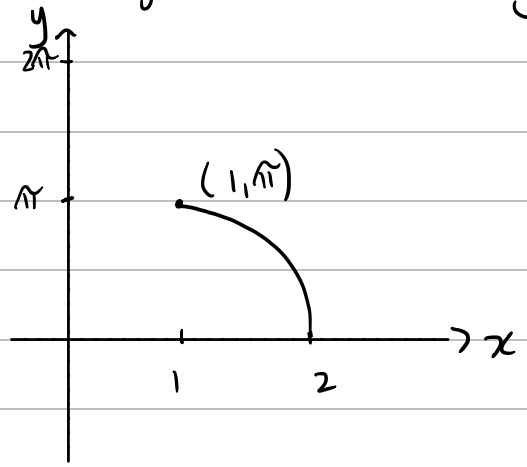
$\therefore$ Domain $f(x)$: $1 \le x \le 2$

Range $\cos^{-1} x$ $0 \le \cos^{-1}(x-1) \le \pi$

$0 \le 2\cos^{-1}(x-1) \le 2\pi$

$\therefore 0 \le y \le 2\pi$

$\therefore$ Range $f(x)$: $0 \le y \le \pi$



1- correct shape and x-intercept at $x=2$

1- coordinates of endpoint at $x=1$.

(ii) $y = 2\cos^{-1}(x-1)$ for $0 \le y \le \pi$

$x = 2\cos^{-1}(y-1)$

$\frac{x}{2} = \cos^{-1}(y-1)$

$\cos\left(\frac{x}{2}\right) = y-1$

$\cos\left(\frac{x}{2}\right)+1 = y$

$\therefore f^{-1}(x) = \cos\left(\frac{x}{2}\right)+1$. $0 \le x \le \pi$

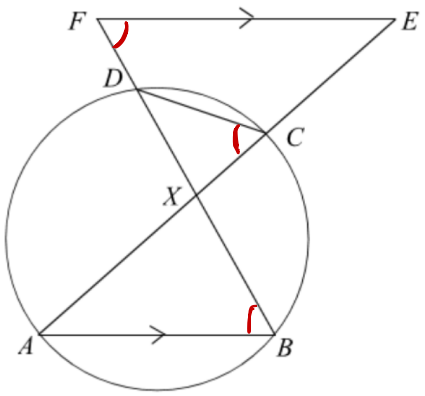
(1) (1)

Question	Suggested solution	Marking guidelines	Markers Notes
(e)	$\frac{d\theta}{dt} = 0.01$		
	$S = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$		
	$\frac{dS}{d\theta} = \frac{1}{2}r^2 - \frac{1}{2}r^2\cos\theta$		
	$= \frac{1}{2}r^2(1-\cos\theta)$		
	when $r=10$ and $\theta = \frac{\pi}{3}$		
	$\frac{dS}{d\theta} = \frac{1}{2}(10)^2\left(1-\cos\frac{\pi}{3}\right)$	1	
	$= 50\left(1-\frac{1}{2}\right)$		
	$= 25$		
	$\frac{dS}{dt} = \frac{dS}{d\theta} \times \frac{d\theta}{dt}$	1	
	$= 25 \times 0.01$		
	$= 0.25$	1	
	$\therefore$ Area increases at $0.25 cm^2 s^{-1}$		

Question	Suggested solution	Marking guidelines	Markers Notes
13.(a)(i)	$f(x) = x^3 + 2x - 7$		
	$f(1) = 1^3 + 2(1) - 7$		
	$= -4$		
	< 0		
	$f(2) = 2^3 + 2(2) - 7$		
	$= 5$		
	> 0	1	
	Since $f(1) < 0$ and $f(2) > 0$		
	there is a root between		
	1 and 2.	1	
(ii)	$f'(x) = 3x^2 + 2$		
	> 0 for all real x		
	$\therefore f(x)$ is monotonic increasing		
	$\Rightarrow$ graph cannot cross x-axis		
	more than once		
	Hence α is the only real root	1	
(iii)	$f(1.5) = 1.5^3 + 2(1.5) - 7$		
	$= -0.625$		
	$f'(1.5) = 3(1.5)^2 + 2$	1	
	$= 8.75$		
	$\alpha_2 = \alpha_1 - \dfrac{f(\alpha_1)}{f'(\alpha_1)}$		
	$= 1.5 - \dfrac{(-0.625)}{8.75}$		
	≈ 1.6 (1dp)	1	

Question	Suggested solution	Marking guidelines	Markers Notes

(b)



$\angle ABD = \angle ACD$ ($\angle$'s in same segment) 1

$\angle ABF = \angle BFE$ (alternate $\angle$'s, FE $\parallel$ AB) 1

$\therefore \angle ACD = \angle DFE$

$\therefore$ DCEF is a cyclic quadrilateral (exterior $\angle$ equal to opposite interior $\angle$) 1

(c) $\int_{\sqrt{2}}^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \left[\sin^{-1}\frac{x}{2}\right]_{\sqrt{2}}^{\sqrt{3}}$ 1

$= \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{\sqrt{2}}{2}$ 1

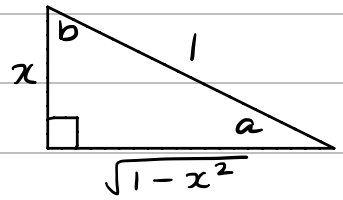
$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{4\pi - 3\pi}{12}$

$= \frac{\pi}{12}$ 1

Question	Suggested solution	Marking guidelines	Markers Notes
(d)(i)	$x^2 = 4y$ $y = \frac{x^2}{4}$ $\frac{dy}{dx} = \frac{x}{2}$ Gradient of tangent at P $= \frac{2t}{2}$ $= t$ $\therefore$ gradient of normal at P is $-\frac{1}{t}$.	1	
	$m_{QP} = \frac{t^2 - \left(t + \frac{2}{t}\right)^2}{2t + 2\left(t + \frac{2}{t}\right)}$ $= \frac{\left(t - \left(t + \frac{2}{t}\right)\right)\left(t + \left(t + \frac{2}{t}\right)\right)}{2\left(t + \left(t + \frac{2}{t}\right)\right)}$ $= \frac{t - t - \frac{2}{t}}{2}$ $= -\frac{2}{t} \div 2$ $= -\frac{2}{t} \times \frac{1}{2}$ $= -\frac{1}{t}$	1	
	$\therefore$ QP is normal to tangent at P		
(ii)	$x = \frac{-2\left(t + \frac{2}{t}\right) + 2t}{2}$ $= -t - \frac{2}{t} + t$ $= -\frac{2}{t}$ $t = -\frac{2}{x}$ ①		

Question	Suggested solution	Marking guidelines	Markers Notes
	$y = \frac{(t+\frac{2}{t})^2 + t^2}{2}$ $= \frac{t^2 + 4 + \frac{4}{t^2} + t^2}{2}$ $= \frac{2t^2 + 4 + \frac{4}{t^2}}{2}$ $= \frac{2(t^2 + 2 + \frac{2}{t^2})}{2}$ $= t^2 + 2 + \frac{2}{t^2}$ ②	1	
	sub ① into ② $y = (-\frac{2}{x})^2 + 2 + \frac{2}{(-\frac{2}{x})^2}$ $= \frac{4}{x^2} + 2 + \frac{2}{(\frac{4}{x^2})}$ $= \frac{4}{x^2} + 2 + \frac{x^2}{2}$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
14(a)	$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{3}$		Alternatively: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
	Let $a = \sin^{-1}x$ $\quad b = \cos^{-1}x$		$\therefore \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$
	$\sin a = x$ $\quad \cos b = x$		$\sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{\pi}{3}$
			$2\sin^{-1}x - \frac{\pi}{2} = \frac{\pi}{3}$
			$2\sin^{-1}x = \frac{5\pi}{6}$
			$\sin^{-1}x = \frac{5\pi}{12}$
	$a - b = \frac{\pi}{3}$ ①		
	$a + b = \frac{\pi}{2}$ ②	1	
	①+② $\quad 2a = \frac{\pi}{3} + \frac{\pi}{2}$		
	$2a = \frac{5\pi}{6}$		
	$a = \frac{5\pi}{12}$		
	$\therefore \sin^{-1}x = \frac{5\pi}{12}$	1	
(b)(i)	$\sin 2t - \sqrt{3}\cos 2t = R\sin(2t - \alpha)$		
	$R = \sqrt{1^2 + (\sqrt{3})^2}$ $\quad \tan\alpha = \frac{\sqrt{3}}{1}$		
	$= \sqrt{1+3}$ $\quad \alpha = \frac{\pi}{3}$	1	
	$= \sqrt{4}$		
	$= 2$		
	$\therefore \sin 2t - \sqrt{3}\cos 2t = 2\sin(2t - \frac{\pi}{3})$	1	
(ii)	$x = 2\sin(2t - \frac{\pi}{3})$		
	$\dot{x} = 4\cos(2t - \frac{\pi}{3})$		
	$\ddot{x} = -8\sin(2t - \frac{\pi}{3})$	1	
	$= -4(2\sin(2t - \frac{\pi}{3})$		
	$= -4x$		
	$= -2^2 x$	1	
	$\therefore$ since acceleration is in the form $-n^2x$, the particle is moving in SHM		

Question	Suggested solution	Marking guidelines	Markers Notes
(iii)	$n = 2$		
	$T = \frac{2\pi}{n}$		
	$= \frac{2\pi}{2}$		
	$= \pi$	1	
(iv)	$2\sin(2t - \frac{\pi}{3}) = 2$		
	$\sin(2t - \frac{\pi}{3}) = 1$		
	$2t - \frac{\pi}{3} = \frac{\pi}{2}$	1	
	$2t = \frac{\pi}{2} + \frac{\pi}{3}$		
	$= \frac{5\pi}{6}$		
	$t = \frac{5\pi}{12}$	1	
	∴ Particle first reached 2 metres right of the origin at $\frac{5\pi}{12}$ seconds		
(c) (i)	$y = 60t\sin\theta - 5t^2$ ①		
	$\frac{dy}{dt} = 60\sin\theta - 10t$		
	Maximum height when $y' = 0$		
	$0 = 60\sin\theta - 10t$		
	$10t = 60\sin\theta$		
	$t = 6\sin\theta$ ②	1	
	sub ② into ①		
	$y = 60(6\sin\theta)\sin\theta - 5(6\sin\theta)^2$		
	$= 360\sin^2\theta - 180\sin^2\theta$		
	$= 180\sin^2\theta$ as required	1	

Question	Suggested solution	Marking guidelines	Markers Notes
14(c)(ii)	$x = 60 + \cos\theta$ ③		
	sub ② into ③		
	$x = 60(6\sin\theta)\cos\theta$		
	$= 360\sin\theta\cos\theta$		
	$= 180(2\sin\theta\cos\theta)$		
	$= 180\sin 2\theta$ as required	1	
(iii)	For best viewing		
	$90 < x < 115$		
	$90 < 180\sin 2\theta < 115$		
	$\frac{1}{2} < \sin 2\theta < \frac{23}{36}$		
	$30° < 2\theta < 40°$ or $140° < 2\theta < 150°$		
	$15° < \theta < 20°$ $70° < \theta < 75°$	1	
	Also $y \geq 140$		
	$180\sin^2\theta \geq 140$		
	$\sin^2\theta \geq \frac{7}{9}$		
	$\theta \geq \sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$		
	$\geq 62°$	1	
	$\therefore 70° < \theta < 75°$	1	