SYDNEY GRAMMAR SCHOOL

NAME

MATHS MASTER

CANDIDATE NUMBER

2023 Term III Annual Examination

Form V Mathematics Extension 1

Thursday 31st August 2023
12:40pm

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 80

Section I (8 marks) Questions 1 – 8

- This section is multiple-choice. Each question is worth 1 mark.
- Record your answers on the provided answer sheet.

Section II (72 marks) Questions 9 – 14

- Relevant mathematical reasoning and calculations are required.
- Start each question in a new booklet.

Collection

- Your name and master should only be written on this page.
- Write your candidate number on this page, on each booklet and on the multiple choice sheet.
- If you use multiple booklets for a question, place them inside the first booklet for the question.
- Arrange your solutions in order.
- Place everything inside this question booklet.

Checklist

- Reference sheet
- Multiple-choice answer sheet
- 6 booklets per boy
- Candidature: 146 pupils

Writer: PC

Section I

Questions in this section are multiple-choice.
Record the single best answer for each question on the provided answer sheet.

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1. What are the asymptotes of $y = \dfrac{2x}{(x-1)(x-3)}$?

   (A) $y = 0, x = -1, x = -3$

   (B) $y = 0, x = 1, x = 3$

   (C) $y = 2, x = -1, x = -3$

   (D) $y = 2, x = 1, x = 3$

2. In which polynomial is the constant term greater than the degree?

   (A) $3x^2 + x + 2$

   (B) $(2x - 3)^3$

   (C) $x\left(5x^2 + 4\right)$

   (D) $(x - 2)^4$

3. What is the coefficient of x^7 in the expansion of $(2x - 3)^9$?

   (A) $-41\,472$

   (B) $41\,472$

   (C) $-314\,928$

   (D) $314\,928$

4. A family of seven is seated randomly around a circular table. What is the probability that the two youngest members of the family are seated together?

   (A) $\dfrac{5!\,2!}{6!}$

   (B) $\dfrac{5!}{2!\,6!}$

   (C) $\dfrac{5!\,2!}{7!}$

   (D) $\dfrac{5!}{2!\,7!}$

5. In each case, when $P(x) = x^3 + m$ or $Q(x) = nx^2 - 11$ is divided by $(x - 2)$, the remainder is 1. What are the values of m and n?

   (A) $m = 7, n = 3$

   (B) $m = 7, n = -3$

   (C) $m = -7, n = 3$

   (D) $m = -7, n = -3$

6. From six people, a team of two and a team of four are to be formed. How many ways can this be done if Alex must be on the team of four?

   (A) ${}^6\mathrm{C}_4 \div 2$

   (B) ${}^5\mathrm{C}_3 \times 2$

   (C) ${}^5\mathrm{C}_3$

   (D) ${}^6\mathrm{C}_4 - {}^6\mathrm{C}_2$

7. Consider a non-zero even function $y = f(x)$ and an odd function $y = g(x)$. Both functions have domain all real x. Which of the following functions must be odd?

   (A) $y = f(x) + g(x)$

   (B) $y = f(x)g(x)$

   (C) $y = f(g(x))$

   (D) $y = g(f(x))$

8. Students choosing a particular course must study 3 subjects, one from each of the 3 columns in the table below. That is, one Science, one Humanity, and one Modern Language must be chosen.

Science	Humanity	Modern Language
Mathematics	History	French
Physics	English	German
Chemistry	Politics	Japanese
Biology		Indonesian
Geology		

   What is the smallest number of students that would guarantee that at least three of them have chosen the same three subjects?

   (A) 25

   (B) 61

   (C) 121

   (D) 181

End of Section I

The paper continues in the next section

Section II

This section consists of long-answer questions.
Marks may be awarded for reasoning and calculations.
Marks may be lost for poor setting out or poor logic.
Start each question in a new booklet.

QUESTION NINE (12 marks) Start a new answer booklet. **Marks**

(a) Consider the polynomial $P(x) = 4x^3 - 12x^2 - x + c$.

(i) Given $(x - 3)$ is a factor of $P(x)$, find the value of c. 1

(ii) Factorise $P(x)$ fully. 2

(b) A curve is defined parametrically by the equations $x = p - \frac{2}{p}$, $y = p^2 + \frac{4}{p^2}$, $p \neq 0$. 2
Find the Cartesian equation of the curve.

(c) The temperature T°C of a bottle of water, t minutes after it has been removed from a refrigerator is given by the formula $T = 20 - 16e^{-kt}$, where k is a positive constant.

(i) Show that the equation above satisfies the differential equation $\frac{dT}{dt} = k(20 - T)$. 1

(ii) Given that $T = 10$, when $t = 8$, find the value of $\frac{dT}{dt}$ when $t = 8$. Give your answer correct to 2 decimal places. 3

(d) Sketch the polynomial $y = (x + 1)^2(x - 1)^3(x - 3)$, labelling all intercepts with the coordinate axes. 3

QUESTION TEN (12 marks) Start a new answer booklet. **Marks**

(a) Express $\left(n+\sqrt{3}\right)^3$ in the form $a+b\sqrt{3}$, where a and b are in terms of n. 2

(b) Prove that: 2

$$\binom{n}{1}^2 = \binom{n}{2} + \binom{n+1}{2}$$

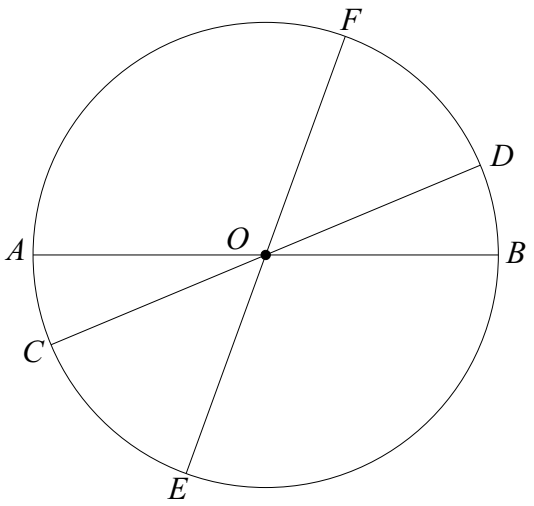
.

(c) Solve the inequalities:

(i) $|2x+1| > 7$ 2

(ii) $\dfrac{x}{x-1} \geq 3$ 3

(d)



In the diagram above AB, CD, and EF are diameters of the circle with centre O.

(i) How many different triangles can be formed by selecting three points from A, B, C, D, E, and F? 1

(ii) How many different triangles can be formed by selecting three points from O, A, B, C, D, E and F? Note: three collinear points do not form a triangle. 1

(iii) How many different quadrilaterals can be formed by selecting four points from O, A, B, C, D, E and F? 1

QUESTION ELEVEN (12 marks) Start a new answer booklet. **Marks**

(a) The polynomial $P(x) = 2x^3 - x^2 + 16x - 8$ has zeroes α, β and γ. Find the value of:

(i) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ 1

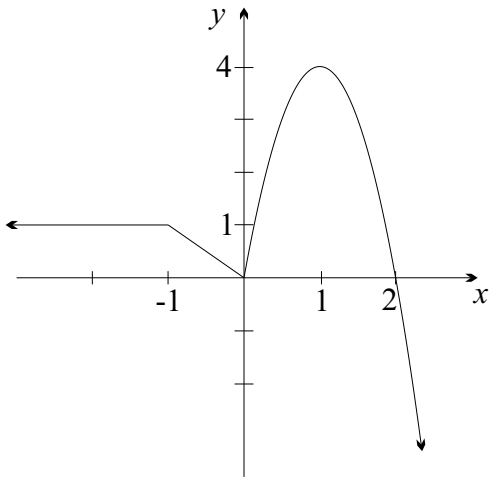
(ii) $(\alpha + 1)(\beta + 1)(\gamma + 1)$ 2

(b) Susan is listening to her playlist which comprises ten songs, exactly three of which are played by her favourite band. When Susan listens to her playlist, all ten songs are played once in random order.

(i) What is the probability that the first three songs played are all by Susan's favourite band? 1

(ii) What is the probability that three of the first five songs played are by Susan's favourite band? 1

(c)



The diagram above shows the graph of $y = f(x)$. Make neat sketches on separate sets of axes of each of the following, showing important features.

(i) $y = f(|x|)$ 1

(ii) $y = \dfrac{1}{f(x)}$ 3

(iii) $y = \sqrt{f(x)}$ 3

QUESTION TWELVE (12 marks) Start a new answer booklet. **Marks**

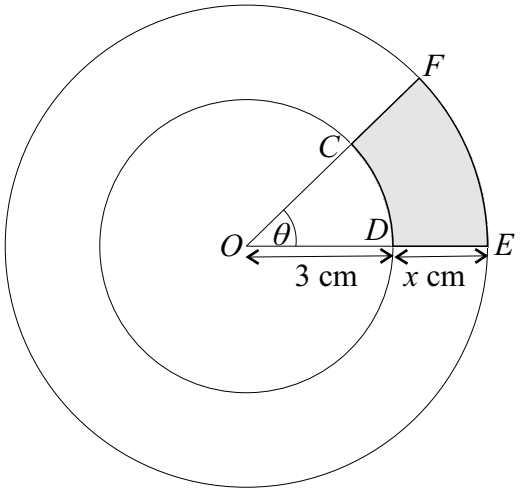
(a) Solve $2\cos^2 x - 3\sin x = 0$, for $0 \leq x \leq 2\pi$. Give your answers in exact form. [3]

(b) How many different arrangements are there of the letters in the word **SEEKERS** if:

(i) all of the **E**'s are together? [1]

(ii) the **S**'s are separate and the **K** is somewhere to the left of the **R**? [2]

(c)



The diagram above shows two concentric circles with common centre O. The smaller circle has radius 3 cm and the larger circle has radius $(x + 3)$ cm. The sectors COD and FOE form an angle θ radians at the centre O.

For the shaded region $CDEF$, let P and A represent its perimeter and area respectively.

(i) Show that $P = (\theta + 2)x + 6\theta$. [1]

(ii) If x is fixed at 5 cm and θ increases at a rate of 0.1 radians per second, find the rate at which P increases. [2]

(iii) If θ is fixed at 1 radian and x increases at a rate of 0.5 cm per second, find the rate at which A is increasing when $x = 5$. [3]

QUESTION THIRTEEN (12 marks) Start a new answer booklet. **Marks**

(a) Find the coefficient of x^2 in the expansion of $\left(1+x+x^2\right)(2+x)^5$. [2]

(b) (i) Given the function $f(x)=\dfrac{kx}{x-k}$, show that $f^{-1}(x)=f(x)$. [2]

(ii) Sketch $y=\dfrac{kx}{x-k}$, where k is a positive constant. Show all asymptotes and intercepts. [2]

(c) Given that the polynomial x^3+px^2+qx+r has zeroes α, β and $\alpha+\beta$, show that [4]

$$4pq = p^3 + 8r.$$

(d) [2]

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1
\end{array}$$

In the diagram above, it can be observed that the six numbers surrounding $\binom{5}{2}$ are linked in the following way:

$$\binom{4}{1}\binom{5}{3}\binom{6}{2} = 4\times 10\times 15 = 6\times 5\times 20 = \binom{4}{2}\binom{5}{1}\binom{6}{3}$$

Prove the more general case for the six numbers surrounding $\binom{n}{k}$:

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$$

QUESTION FOURTEEN (12 marks) Start a new answer booklet. **Marks**

(a) Given that $y = \left(x + \sqrt{1+x^2}\right)^n$, show that $\dfrac{dy}{dx} = \dfrac{ny}{\sqrt{1+x^2}}$. 2

(b) Solve the inequality: $2|x-1| > \sqrt{x(4-x)}$. 3

(c) (i) For any two distinct integers p and q, explain why $p^3q - pq^3$ is an even integer. 1

(ii) Using the Pigeonhole Principle, or otherwise, prove that for any three distinct positive integers, one can always choose two of them so that $p^3q - pq^3$ is divisible by 10. 2

(d) Consider the polynomial $P(x) = x^3 + 3ax^2 + 3bx + c$.

(i) If $P(x) = 0$ has a repeated root, show that this root also satisfies the quadratic equation 1

$$x^2 + 2ax + b = 0\,.$$

(ii) Furthermore, show that if $P(x) = 0$ has a repeated root, the repeated root is 3

$$\frac{c - ab}{2(a^2 - b)}\,.$$

END OF PAPER

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- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

B L A N K P A G E