Name: _____

Teacher: _____

St George Girls High School

Mathematics Extension 1

2022 Year 11 End of Course Examination

General Instructions

- Reading Time: **10 minutes**
- Working time: **90 minutes**
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I,** use the multiple-choice answer sheet provided
- For questions in **Section II:**
  - Answer the questions in the writing booklets provided
  - Begin each question in a separate writing booklet
  - Extra writing booklets are available if required
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 59

Section I – 4 marks (pages 3 – 4)

- Attempt Questions 1 – 4
- Allow about 6 minutes for this section

Section II – 55 marks (pages 5 – 9)

- Attempt all questions
- Allow about 1 hour and 24 minutes for this section

Q1 – 4	/4
Q5	/11
Q6	/11
Q7	/11
Q8	/11
Q9	/11
Total	**/59**
	%

Section I

4 marks
Attempt Questions 1 – 4
Allow about 6 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1 to 4.

1. Which of the following is **not** a polynomial?

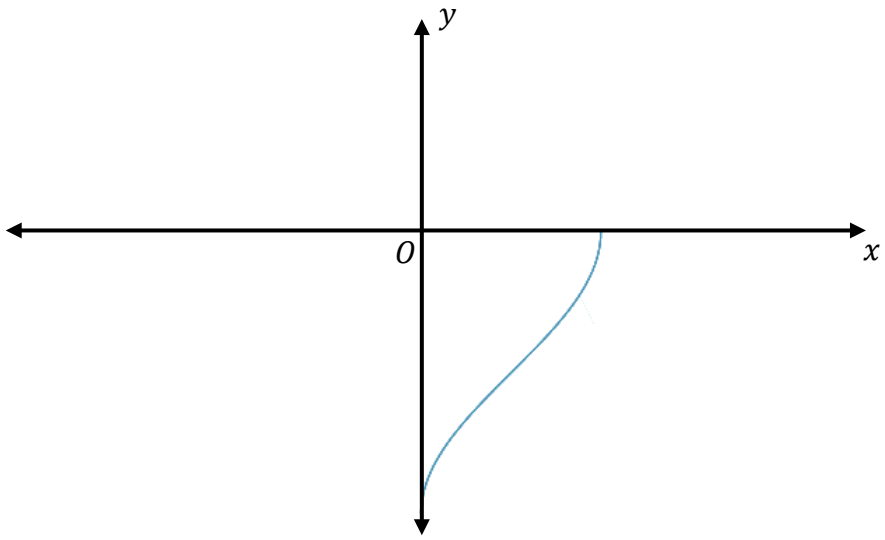
(A) $2x^3 + x^{-1}$

(B) -17π

(C) $3x^2 - 2x + 5^{-\frac{1}{3}}$

(D) $\sqrt{3}x^4 + x - 14$

2. The graph of $y = \sin^{-1}(x + a) + b$ is shown below. Find the values of a and b.



(A) $a = 1, b = -\pi$

(B) $a = 1, b = -\frac{\pi}{2}$

(C) $a = -1, b = -\frac{\pi}{2}$

(D) $a = -1, b = -\pi$

3. The exact value of $\tan 165°$ is:

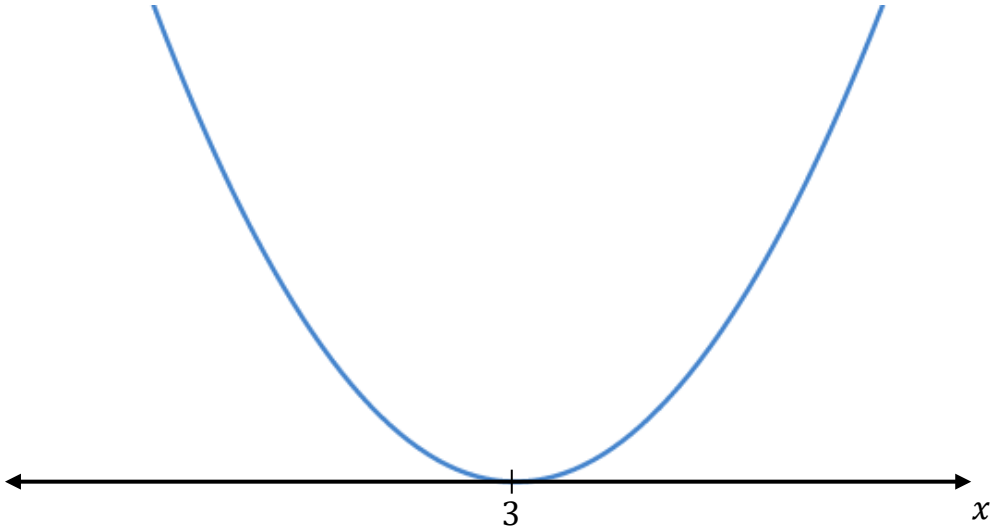
(A) $\tan 90° + \tan 75°$

(B) $\pm(2 - \sqrt{3})$

(C) $\frac{2\tan 15°}{1-\tan^2 15°}$

(D) $\sqrt{3} - 2$

4. Part of the graph of $y = P(x)$ is shown below.



Which of the following could be the equation of $P(x)$?

(A) $P(x) = (x-4)(x+3)^2(x-3)^2$

(B) $P(x) = (x-4)(x+3)^3(x-3)^3$

(C) $P(x) = -(x-4)(x+3)^2(x-3)^2$

(D) $P(x) = -(x-4)(x+3)^3(x-3)^3$

End of Section I

Section II 55 marks

Attempt Questions 5 - 9
Allow about 1 hour and 24 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 5 – 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (11 marks) Begin a new writing booklet. **Marks**

a) For the polynomial $P(x) = 8x - 3x^4 + 11$, state:

i) the degree. 1

ii) the leading coefficient. 1

iii) one of the zeroes. 1

b) For the function $\{(1,2), (3,-1), (4,2), (-1,0)\}$:

i) Classify the function as one-to-one, one-to-many, many-to-one, or many-to-many. 1

ii) State the inverse relation. 1

iii) Is this inverse a function? Justify your answer. 1

c) Solve:

i) $|x - 5| < 3$ 2

ii) $x \geq \frac{7}{x}$ 3

Question 6 (11 marks) Begin a new writing booklet. **Marks**

a) Use the t-formulae to solve $3\sin x - \cos x = 1$, for $0 \leq x \leq 2\pi$. 3

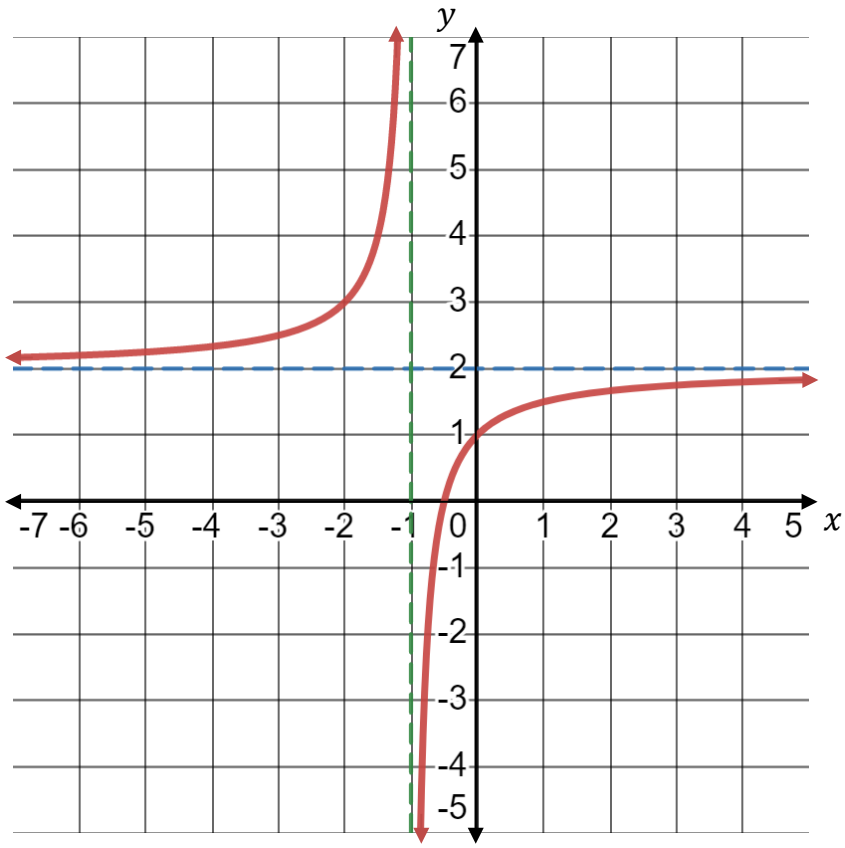
b) Let α, β, and γ be the roots of $2x^3 + 3x - 4 = 0$. Find, in simplest form:

i) $\alpha + \beta + \gamma$ 1

ii) $\alpha\beta + \beta\gamma + \gamma\alpha$ 1

iii) $\alpha^2 + \alpha\beta + \alpha\gamma + 2\alpha\beta + 2\beta^2 + 2\beta\gamma$ 2

c) The graph of $y = f(x)$ is shown below.



Draw neat sketches of at least one-third of a page of:

i) $y = f(|x|)$ 2

ii) $y = \sqrt{f(x)}$ 2

Question 7 (11 marks) Begin a new writing booklet. **Marks**

a) Sketch $y = 2e^{-x} + 1$, showing all important features. 3

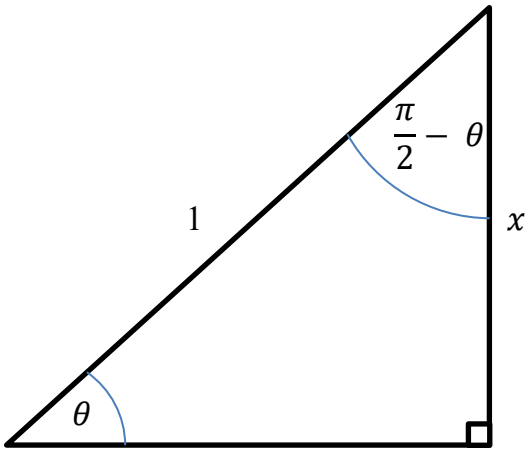
b) The displacement of a particle as it moves along the number line is given by $x = e^t - et$, where x is in metres and t is in seconds.

i) Find the initial displacement. 1

ii) Find when and where the particle is at rest. 3

iii) Find the exact distance travelled in the first 2 seconds. 2

c) Using the triangle below, or otherwise, prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$. 2



Question 8 (11 marks) Begin a new writing booklet. **Marks**

a) The remainders when $P(x)$ is divided by $(x-3)$ and $(x+1)$ are 4 and -4 respectively. Find the remainder in the form $R(x) = ax + b$ when $P(x)$ is divided by $x^2 - 2x - 3$. 3

b) Consider the function $f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9$.

i) Show that $f(x)$ has a root of multiplicity 2 at $x = 3$. 2

ii) Prove that $x = 3$ is the only point where the graph meets the x-axis, clearly justifying your response. 2

c) The rate of growth of a sourdough starter is proportional to the difference between its volume, V, and the size of the jar it lives in. The volume of the starter can be modelled by $V = 250 - 240e^{-kt}$, where V is in cm^3, t is in hours, and k is a positive constant.

i) Show that the rate of growth of the volume of sourdough starter is given by the differential equation $\dfrac{dV}{dt} = k(250 - V)$. 1

ii) After 8 hours, the volume is 130cm^3. Find k, correct to 3 decimal places. 2

iii) How big is the jar? 1

Question 9 (11 marks) Begin a new writing booklet. **Marks**

a) Consider the function $f(x) = \dfrac{e^x + e^2}{e^2}$.

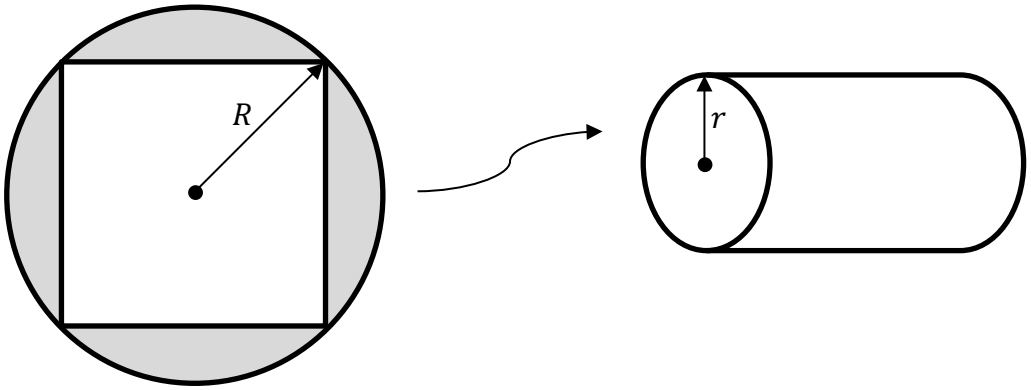
i) Find the domain in which $f(x)$ is increasing. 2

ii) Find $f^{-1}(x)$, the inverse function. 2

iii) Find the equation of the tangent to $y = f(x)$ when $x = 2$. 1

iv) Hence or otherwise solve $f(x) = f^{-1}(x)$. 1

b) A burrito is a delicious wrap, made by enclosing a range of fillings in a rolled-up circular tortilla. The fillings are placed towards the centre of the tortilla, which is then rolled to form the cylindrical burrito. The four segments around the edges (shaded in the diagram below) are folded into the cylinder as part of the rolling process. The dimensions of the final burrito come from the largest square that fits inside the tortilla, as shown in the diagram.



i) Show that the volume of the burrito is given by $V = \dfrac{R^3\sqrt{2}}{2\pi}$ where R is the radius of the tortilla. 3

ii) Before making the burrito, the tortilla needs to be cooked, during which time the radius of the tortilla shrinks by 3cm/min. Find the rate at which the volume of the potential burrito is decreasing when the radius of the tortilla is 20cm. 2

End of Examination

Name: _____

Teacher:_____

Section I

Year 11

Mathematics Extension 1

End of Course Examination

2022

Multiple-choice Answer Sheet - Questions 1 – 4

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○