
Student Number

SCEGGS Darlinghurst

2022 **PRELIMINARY COURSE SEMESTER 2 EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 1 hour and 30 minutes
- Write using black pen
- Write your student number at the top of this page
- NESA approved calculators may be used
- A Reference Sheet is provided on a separate sheet
- For Questions 8-10, show relevant mathematical reasoning and/or calculations

Total marks: 50

Section I – Multiple Choice – 7 marks

- Attempt Questions 1–7
- Allow about 12 minutes for this section

Section II – 43 marks

- Attempt Questions 8-10
- Allow about 1 hour and 18 minutes for this section

Section	Questions	Possible Marks	Awarded Marks
I Multiple Choice	1-7	7	
II Short Answers	8-10	43	
TOTAL		50	

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

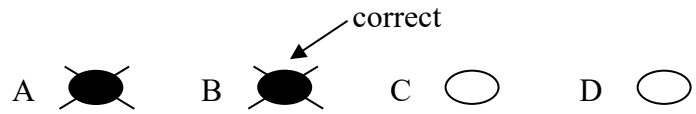
Sample: 2 + 4 = A. 2 B. 6 C. 8 D. 9



If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



Section I

7 marks
Attempt Questions 1–7
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1–7.

---

1 The polynomial $P(x)$ has degree 3.

What is the degree of $(P(x))^2$?

A. 5

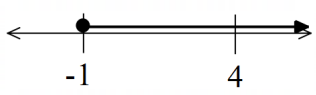
B. 9

C. 6

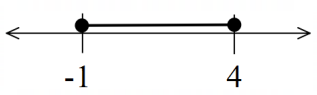
D. 8

2 Which diagram best represents the set of solutions of the inequality $|2x-3| \geq 5$?

A.



B.



C.



D.



3 If $\sin\theta = \frac{1}{\sqrt{3}}$, and $\cos\theta < 0$, which of the following is the exact value of $\tan 2\theta$?

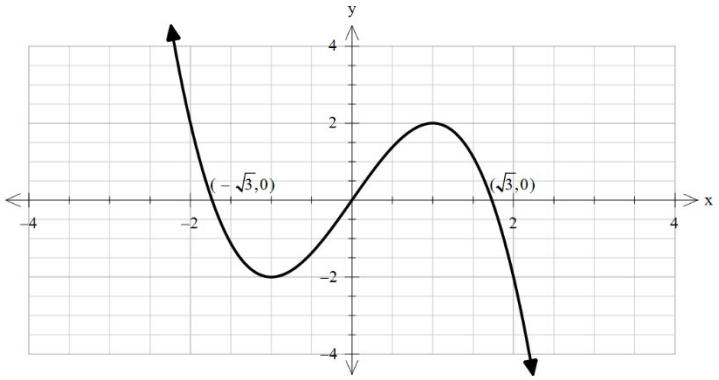
A. $-2\sqrt{2}$

B. $\frac{-1}{2\sqrt{2}}$

C. $\frac{1}{2\sqrt{2}}$

D. $2\sqrt{2}$

4 The curve $f(x) = 3x - x^3$ is shown below.



What is the largest domain containing the origin for which $f(x)$ has an inverse function?

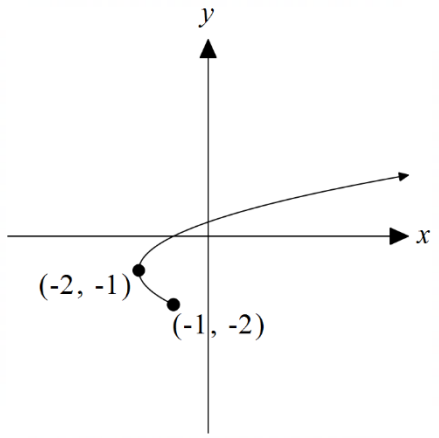
A. $[-1,1]$

B. $[-\sqrt{3},\sqrt{3}]$

C. $[-2,2]$

D. $[0,2]$

5 A parabola is shown in the graph below.



Which pair of parametric equations best represents the graph?

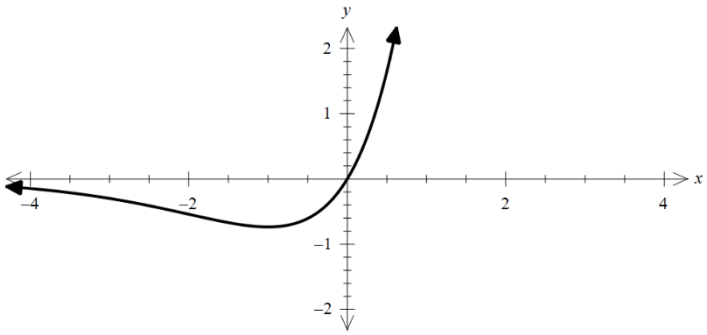
A. $\begin{cases} x = 2 + t^2 \\ y = 1 + t \end{cases}, t \geq -1$

B. $\begin{cases} x = -2 + t^2 \\ y = -1 + t \end{cases}, t \geq -1$

C. $\begin{cases} x = 2 + t^2 \\ y = 1 + t \end{cases}, t \leq -1$

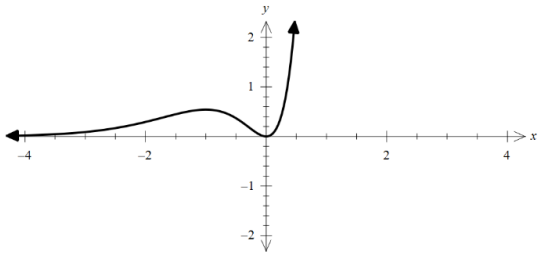
D. $\begin{cases} x = -2 + t^2 \\ y = -1 + t \end{cases}, t \leq -1$

6 The graph of the function $f(x)$ is shown below.

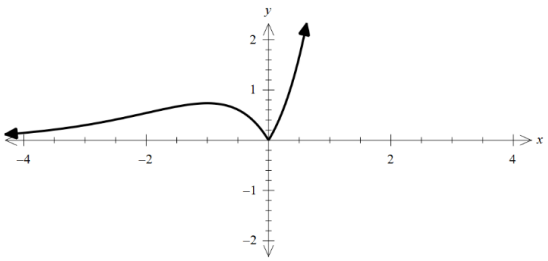


Which of the following graphs best represents $[f(x)]^2$?

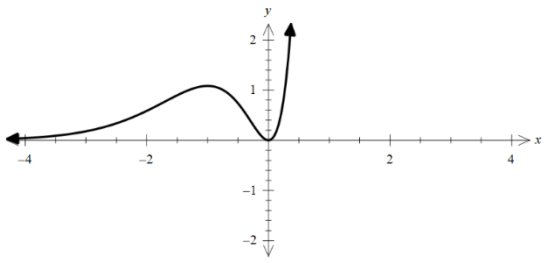
A.



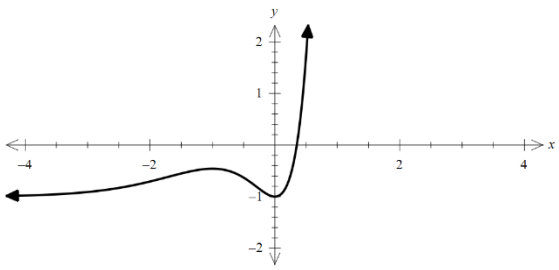
B.



C.

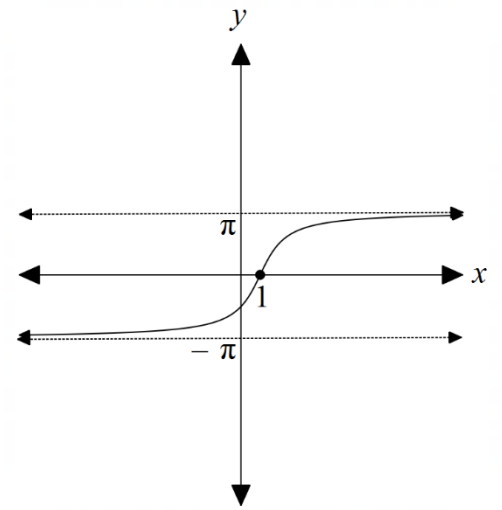


D.

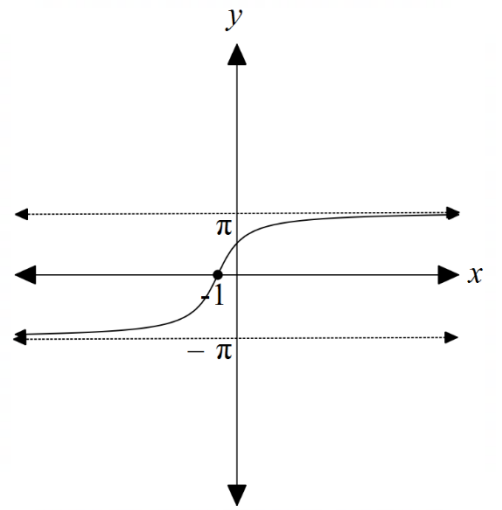


7 Which graph best represents $y = 2tan^{-1}(1-x)$?

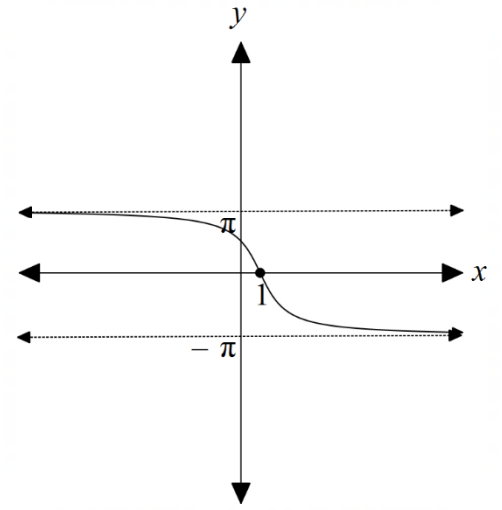
A.



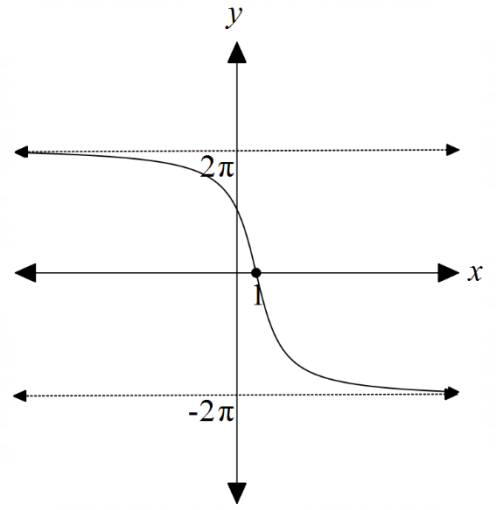
B.



C.



D.



End of Section I

BLANK PAGE

Section II

43 marks
Attempt Questions 8-10
Allow about 1 hour and 18 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

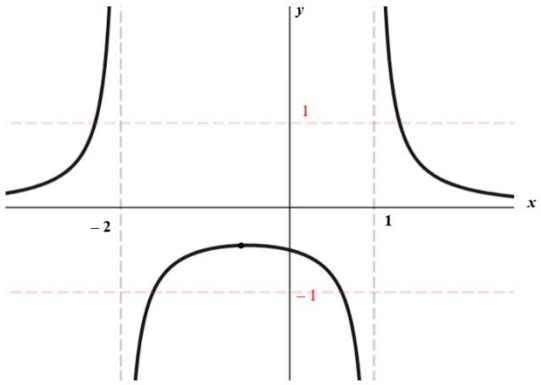
Question 8 (15 marks)

(a) Solve $2x^2 + 7x - 15 \leq 0.$ 2

(b) What is the exact value of $\cos(\tan^{-1}(-3))$? 2

(c) A polynomial $P(x)$ has a remainder of $2x+1$ when divided by $x^2 - 5x + 6.$ 2
What is the remainder when $P(x)$ is divided by $x-3$?

(d) The graph of $y = f(x)$ is shown below.



Use the separate sheet provided to draw the graphs of :

(i) $y = f\left(|x|\right)$ 2

(ii) $y = \dfrac{1}{f(x)}$ 2

Question 8 continues on page 8

Question 8 (continued)

(e) Prove that $\cos\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)=-\frac{7}{25}.$ 2

(f) Given that α,β,γ are the roots of the equation $2x^3-6x^2+10x-5=0,$ find the value of :

(i) $\alpha+\beta+\gamma$ 1

(ii) $\alpha^2+\beta^2+\gamma^2$ 2

End of Question 8

Question 9 (14 marks) Start a new answer booklet.

(a) Solve $|3x-7| \geq 4.$ 2

(b) Find the exact value of $\cos 75^\circ.$ 2

(c) Solve $\dfrac{x^2+6}{x} \geq 5.$ 3

(d) It is given that $P(x) = x^4 - 5x^3 + ax^2 + bx - 48,$ where a and b are constants, and that $x = 2$ is a double root of $P(x).$

(i) Find the values of a and b. 3

(ii) Factorise $P(x)$ completely into linear factors. 2

(e) Show that $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta.$ 2

End of Question 9

Question 10 (14 marks) Start a new answer booklet.

(a) By letting $t = \tan\frac{\theta}{2}$, find the exact value of $\tan 67.5^{\circ}$. 3

(b) Prove that $4\sin 7\alpha \cos 5\alpha \cos 3\alpha = \sin 15\alpha + \sin 9\alpha + \sin 5\alpha - \sin \alpha$. 3

(c) Consider the function $f(x) = \dfrac{e^{-x}}{1+e^{-x}}$.

(i) Find the inverse $f^{-1}(x)$, of $f(x)$. 2

(ii) Find the domain and range of $f^{-1}(x)$. 2

(d) The position of a particle moving along the x-axis is given by $x = 100e^{-0.2t} + 20t - 80$, where t is the time in seconds and x is the displacement from the origin in metres.

(i) Find the equations for the velocity and the acceleration of the particle. 2

(ii) Find the limiting velocity of the particle. 1

(iii) Show that the particle is always speeding up for any time (t seconds). 1

End of paper