SYDNEY GRAMMAR SCHOOL

NAME _____ MATHEMATICS MASTER _____

2021 Annual Examination

Form V Mathematics Extension 1

Monday 30th August 2021

8:39am

General Instructions

- Working time — 2 hours
- Attempt all questions.
- Write using black pen.
- Calculators approved by NESA may be used.
- A loose reference sheet is provided separate to this paper.

Total Marks: 80

Questions 1 – 6

- Because of the nature of this task, greater weight than normal will be placed on working. Clear reasoning and full calculations are required.
- Record your answers on the writing paper provided.
- Start each question on a new page.
- Write your name and master on each page.

Your sheets must be ORDERED
then scanned and uploaded in a
SINGLE PDF FILE
to the Schoology page of your mathematics class

5A: WJM	5B: SDP	5C: LYL
5D: NJL	5E: FMW	5F: RPL
5G: BLR	5H: CWG	5I: AEA

Checklist

- Reference Sheet
- Loose writing paper

Writer: WJM

QUESTION ONE (13 marks) Start a new page. **Marks**

(a) Solve the inequality $x^2 - 2x - 3 > 0$. [2]

(b) Expand and simplify $(2x + y)^4$. [2]

(c) Consider the function $f(x) = \sqrt{x - 1}$.

(i) Find the equation of the inverse function $f^{-1}(x)$. [1]

(ii) Write down the domain of $f(x)$, and hence state the range of $f^{-1}(x)$. [1]

(d) A polynomial is given by $P(x) = x^3 + 2x^2 - 11x - 12$.

(i) Show that $(x + 1)$ is a factor of $P(x)$. [1]

(ii) Hence fully factorise $P(x)$. [2]

(iii) Sketch the graph of $y = P(x)$, clearly showing any intercepts with the axes. [2]

(e) A 4-digit PIN is to be created using the digits from 0 to 9, where repetition is allowed. How many different PINs can be created that start with a 5 and **do not** contain the number 7? [2]

QUESTION TWO (13 marks) Start a new page. **Marks**

(a) A particular family consists of a mother, a father, and three children. They are to be seated in a car that has a driver's seat, a front passenger's seat, and three seats in the back. How many ways can this be done if:

(i) the mother must drive, and the others can sit in any seat? 1

(ii) an adult must drive and the other adult must sit between two children in the back? 2

(b) The polynomial equation $x^3 + 9x^2 - 2x - 24 = 0$ has roots α, β, and γ.

Find the value of:

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$ 2

(ii) $\alpha^2 + \beta^2 + \gamma^2$ 2

(c) Use a binomial expansion to find integers a and b such that $\left(1 - \sqrt{2}\right)^5 = a + b\sqrt{2}$. 3

(d) The polynomial $P(x) = x^3 + 6x^2 + cx - 24$ has three real zeros. One of which is the average of the other two, so the zeros can be written as α, β, and $\frac{\alpha+\beta}{2}$. Find the zeros of $P(x)$. 3

QUESTION THREE (14 marks) Start a new page. **Marks**

(a) The letters of the word VEGEMITE are rearranged. In how many ways can this be done if:

(i) there are no restrictions? 1

(ii) the word starts with V and ends with T? 1

(iii) the I is directly between the G and the M? 1

(iv) the V is further left than any E? 2

(b) A function is defined parametrically by $x = \dfrac{t-1}{2}$, $y = \dfrac{t+1}{2}$ for $-3 \leq t \leq 3$.

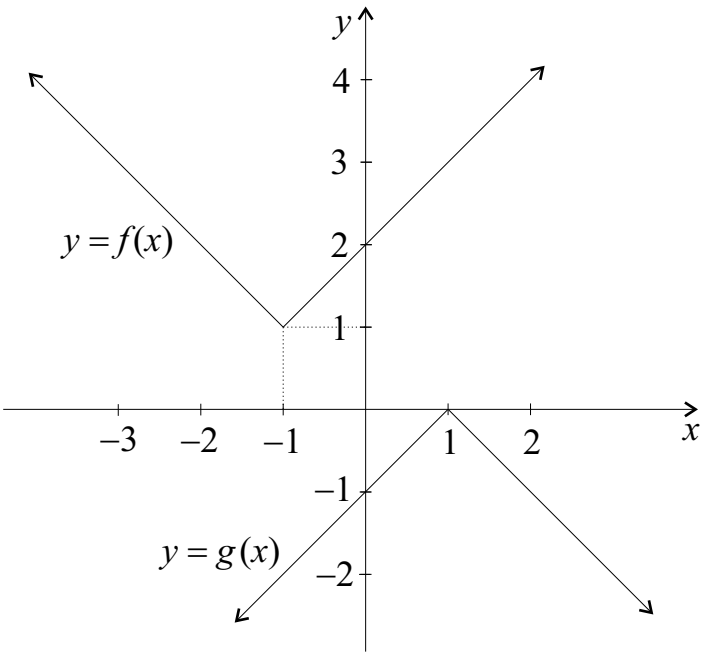
(i) Find the Cartesian equation of the function. 1

(ii) Sketch the function clearly indicating the domain and range, and showing any intercepts with the axes. 2

Question Three continues on the following page.

QUESTION THREE (Continued)

(c) (i)



The diagram above shows the graphs of the functions $y = f(x)$ and $y = g(x)$.

For EACH question below, copy the diagram onto your page. Each graph should be approximately one third of a page. You should include the coordinates of at least three points on each of your graphs.

Sketch the graph of:

(α) $y = f(x) + g(x)$ 2

(β) $y = f(x) - g(x)$ 2

(ii) For the functions $f(x)$ and $g(x)$ in part (c)(i) above, 2

$$f(x) = ag(x + b) + c$$

for some constants a, b and c. Find the values of a, b and c.

QUESTION FOUR (13 marks) Start a new page. **Marks**

(a) A committee of 5 people is to be randomly chosen from 5 women and 4 men. What is the probability that the majority of the committee will be women? 3

(b) The polynomial $P(x) = 2x^3 + 6x^2 + cx + d$ has a triple zero α.

(i) Find the value of α. 2

(ii) Find the values of c and d. 2

(c) A menu at a restaurant contains a choice of 4 entrées, 3 main courses, and 2 desserts. Each customer is required to pick one entrée, one main course, and one dessert. What is the minimum number of customers that the restaurant would have to serve in order to ensure that at least five customers made the same entrée, main, and dessert combination from the menu? Justify your answer. 2

(d) Solve the inequality $\dfrac{6}{x-1} \geq x + 4$. 4

QUESTION FIVE (13 marks) Start a new page. **Marks**

(a) (i) Expand and simplify $(\sin\theta + \cos\theta)^2$. 1

(ii) Hence simplify $\dfrac{\cos\theta - \tan\theta\sin\theta}{1 + 2\sin\theta\cos\theta} + \dfrac{\tan\theta}{\sin\theta + \cos\theta}$. 2

(b) Twelve pupils are taking a class that requires them to complete the same group project. They are to be split into three groups, with each group consisting of four pupils. In calculating the number of ways this can be done, one of the pupils suggests: 2

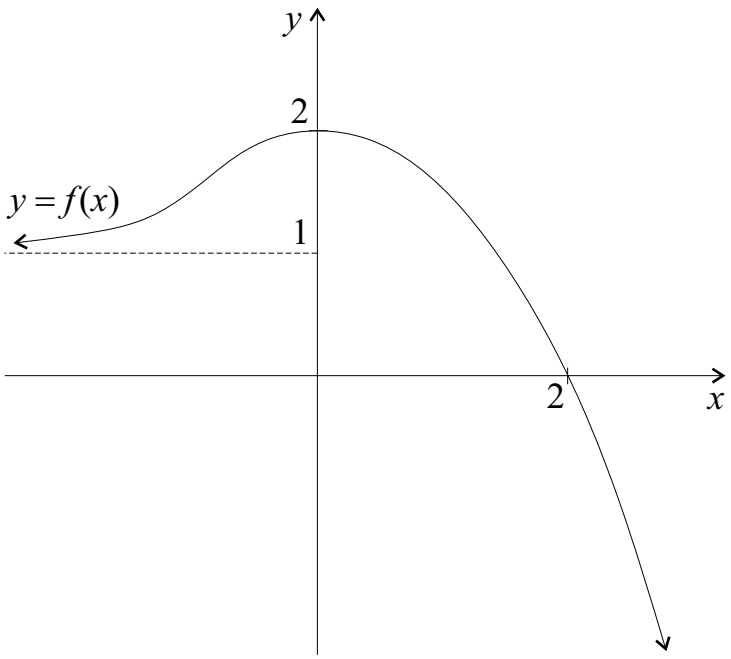
"The first group can be selected in ${}^{12}\mathrm{C}_4$ ways.
The second group can be selected in ${}^{8}\mathrm{C}_4$ ways.
The third group can then be selected in ${}^{4}\mathrm{C}_4$ ways.
Therefore there are ${}^{12}\mathrm{C}_4 \times {}^{8}\mathrm{C}_4 \times {}^{4}\mathrm{C}_4 = 34\,650$ ways we can be split into three groups of four."

Explain why this reasoning is incorrect, and calculate the correct number of ways that the twelve pupils can be split into three groups of four.

Questions Five continues on the following page.

QUESTION FIVE (Continued)

(c)



The diagram above shows the graph of $y = f(x)$, which has an x intercept of 2 and a y intercept of 2. The line $y = 1$ is a horizontal asymptote in the negative direction.

For EACH question below, copy the diagram onto your page. Each graph should be approximately one third of a page.

Sketch the graph of:

(i) $y = \dfrac{1}{f(x)}$ 2

(ii) $|y| = f(-x)$ 3

(iii) $y = xf(x)$ 3

QUESTION SIX (14 marks) Start a new page. **Marks**

(a) In the expansion of $(2+x)^n$, the coefficients of x^5 and x^6 are equal. Find the value of n. [3]

(b) Let $y = f(x)$ and $y = g(x)$ both be continuous, differentiable functions with domain all real x.

When $x = a$, it is known that:

- the line $l_1 : y = m_1x + b_1$, where $m_1 \neq 0$, is a tangent to $y = f(x)$,
- the line $l_2 : y = m_2x + b_2$, where $m_2 \neq 0$, is a tangent to $y = g(x)$, and
- the graph of $y = \dfrac{f(x)}{g(x)}$ is stationary.

(i) Show that $\dfrac{b_1}{m_1} = \dfrac{b_2}{m_2}$. [3]

(ii) Hence show that the lines l_1 and l_2 must intersect on the x-axis. [2]

(c) Horace owns n unique hats, n unique scarves, and n unique vests. Every day, he randomly selects a hat, a scarf, and a vest.

(i) How many hat, scarf, and vest combinations can Horace create? [1]

(ii) All of Horace's hats, scarves, and vests are mixed in one big drawer. [2]

With reference to Horace's clothing situation, explain why

$$\binom{3n}{3} - 3\binom{n}{3} - 6n\binom{n}{2} = n^3.$$

(iii) Horace randomly pulls three items out of his drawer in the hope of getting a hat, scarf, and vest combination. He doesn't know how many items of clothing are in his drawer, but after years of experience he has estimated that there is a $\frac{49}{205}$ chance that he gets a hat, scarf, and vest combination on the first try. [3]

If Horace's estimation is correct, how many items of clothing are in his drawer?

END OF PAPER