Section 1

8 marks
Attempt Questions 1–8.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 8.

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1. Which of the following is an equivalent expression for $\dfrac{\tan(x-y)+\tan y}{1-\tan(x-y)\tan y}$?

(A) $\tan x$

(B) $\tan(x-y)$

(C) $\tan(x-2y)$

(D) $\dfrac{2\tan x-\tan y}{1-\tan x\tan y+\tan^2 y}$

2. How many ways can the letters of the word ISOSCELES be arranged in a line?

(A) 3 024

(B) 30 240

(C) 90 720

(D) 362 880

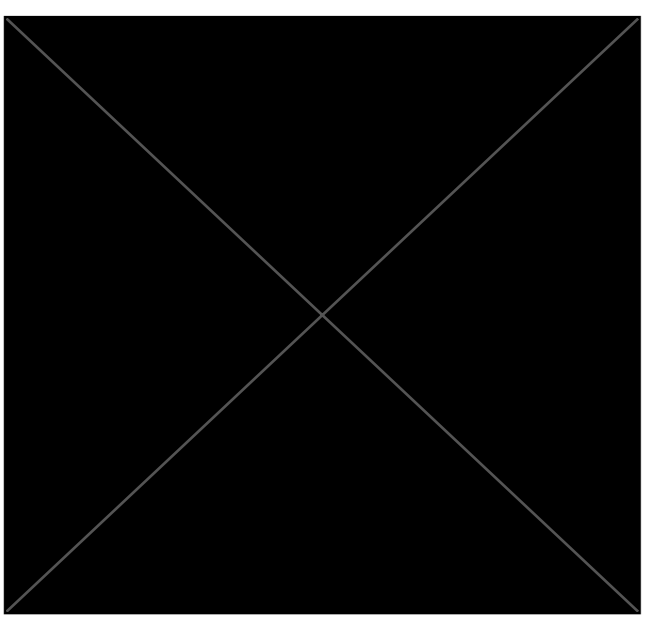
3. What is the value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$?

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{\pi}{6}$

(D) $\frac{5\pi}{6}$



4. What is the value of $\sin^{-1} x + \cos^{-1} x$?

(A) 0

(B) 1

(C) $\frac{\pi}{2}$

(D) π

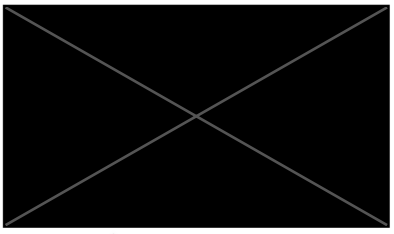
5. Adam has 5 coloured blocks; one red, one blue, one green, one yellow and one white. He stacks two or three blocks on top of each other to form a vertical tower. How many different towers can he form?

(A) 20

(B) 80

(C) 100

(D) 1 200



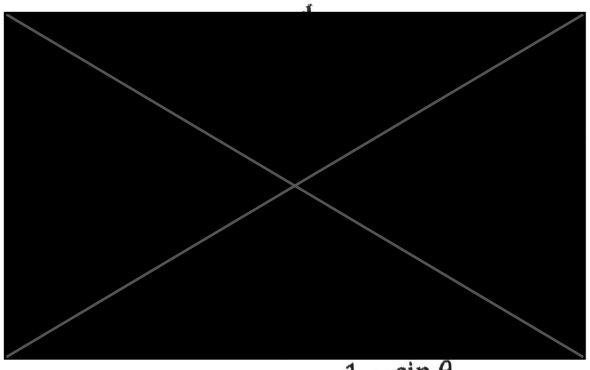
6. The velocity of a particle is given by $v = 2te^{-t}$ ms^{-1}. What is the acceleration at $t = 1$?

(A) $-\frac{2}{e}$ ms^{-2}

(B) 0 ms^{-2}

(C) $\frac{2}{e}$ ms^{-2}

(D) $\frac{4}{e}$ ms^{-2}



7. Given that $t = \tan\frac{\theta}{2}$, which of the following is equivalent to $\frac{1-\sin\theta}{\cos\theta}$?

(A) $-t$

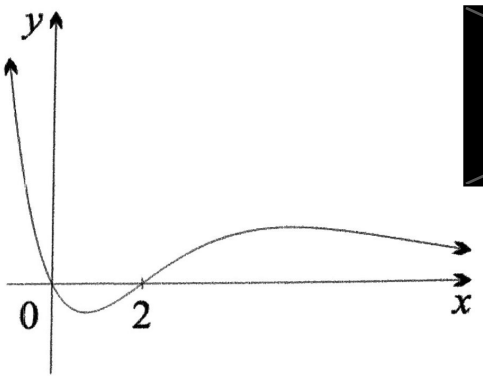
(B) $-\frac{1}{1+t}$

(C) $\frac{t-1}{t+1}$

(D) $\frac{1-t}{1+t}$

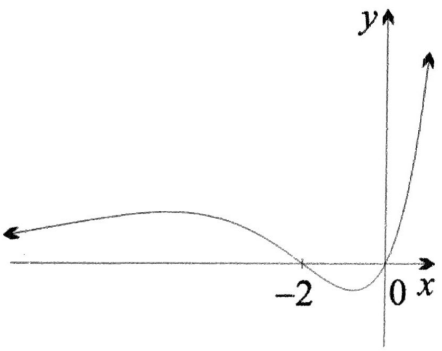


8. The graph of $y = f(x)$ is shown below, with the x axis as a horizontal asymptote for large positive values of x.

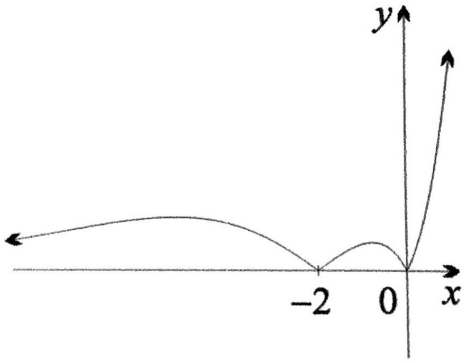


Which of the following best represents $y = |f(-x)|$?

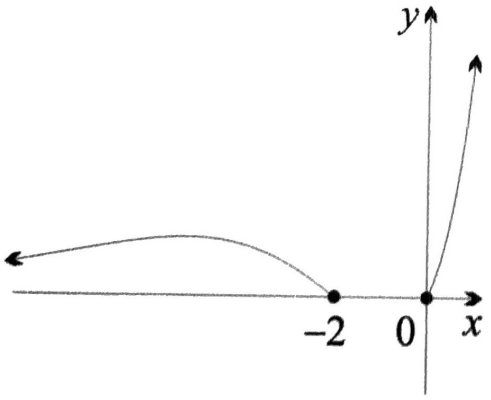
(A)



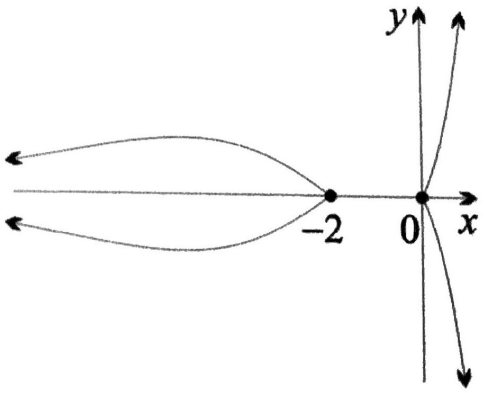
(B)



(C)



(D)



Section II

43 marks
Attempt Questions 9 - 11.
Allow about 1 hour and 15 minutes for this section.

Start each question on a NEW page and put your **student number on EACH page.**
In Questions 9-11, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 (14 marks) Start a NEW Page

(a) Eight students are to be arranged on a circular table at their formal. **2**
How many ways can they be seated if Adam, Ben and Charlie must sit together?

(b) A polynomial $P(x) = (x-1)(x-2)Q(x) + ax + b$, where $Q(x)$ is a polynomial and a and b are constants. **3**
If $(x-2)$ is a factor of $P(x)$ and the remainder when $P(x)$ is divided by $(x-1)$ is 6, find the remainder when $P(x)$ is divided by $x^2 - 3x + 2$. [redacted]

(c) A committee of 4 people is to be chosen from 8 males and 6 females.

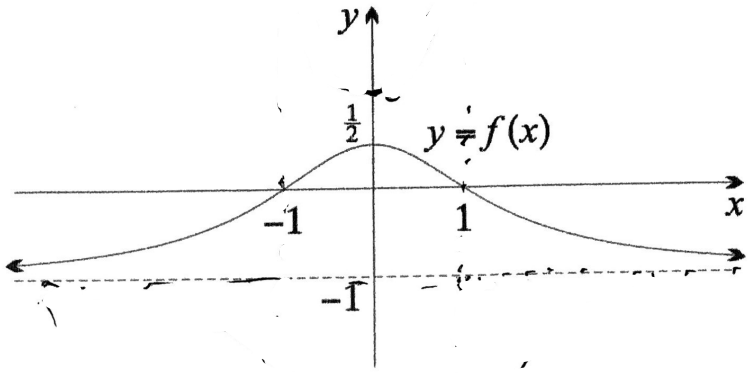
(i) How many ways can this committee be formed if there are no restrictions? **1**

(ii) How many ways can the committee be formed if a majority of the committee members are females? **2**

(d) The function $f(x) = x^2 - 2x$, where $x \geq 1$, has an inverse function. **3**
Find the equation of the inverse function $f^{-1}(x)$.

Question 9 continues over the page

(e) Consider the graph of $y = f(x)$ below, where the $x = -1$ is a horizontal asymptote. **3**



Sketch $y = \frac{1}{f(x)}$, labelling any important features.

Ensure that your diagram is at least ONE-THIRD of the page.

End of Question 9

Question 10 (15 marks) Start a NEW page

(a) Sketch the curve $y = \frac{\pi}{4} + \tan^{-1} x$, labelling any intercepts. **2**

(b) (i) Using the binomial theorem, obtain an expression for $(1+x)^{2n} + (1-x)^{2n}$. **2**

(ii) Hence, show that $1 + \binom{20}{2} + \binom{20}{4} + \cdots + \binom{20}{20} = 2^{19}$. **1**

(c) By showing working, find the exact value of $\cos\left(\cos^{-1}\frac{2}{3} + \tan^{-1} 2\right)$. **3**

(d) A curve has parametric equations given by: **3**

$$x = 1 + \sin\theta$$
$$y = \cos 2\theta - 1$$

Show that the curve is a parabola and explain why the curve has domain $0 \leq x \leq 2$.

(e) (i) Show that $(x+y)(x^2 - xy + y^2) = x^3 + y^3$. **1**

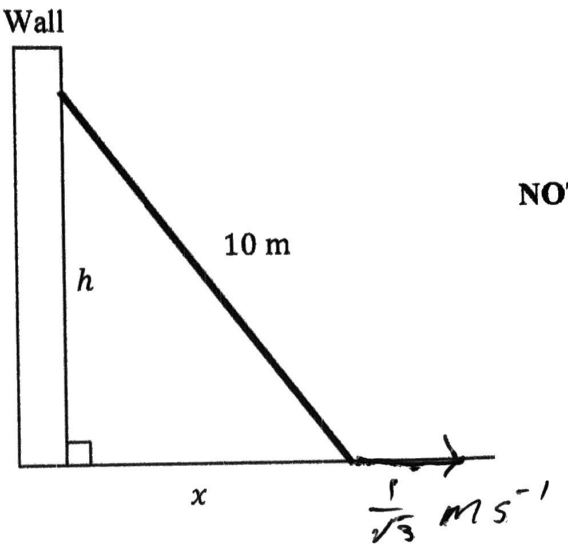
(ii) Hence, prove the trigonometric identity: **3**

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \frac{1}{2}\sin 2\theta = 1.$$

End of Question 10

Question 11 (14 marks) Start a NEW page

(a) The diagram below shows a 10 metre ladder leaning against a wall at h metres above the ground. The bottom of the ladder is x metres from the base of the wall.



NOT TO SCALE

(i) Write an expression for h in terms of x. **1**

(ii) The bottom of the ladder slides away from the wall at a rate of $\frac{1}{\sqrt{3}}$ ms^{-1}. **3**

Find the exact rate at which the top of the ladder is sliding down the wall when the bottom of the ladder is 5 metres from the base of the wall.

(b) Prove that the equation $2x^3 + 5x^2 - 4x - 12 = 0$ has only one root of multiplicity 2. **3**

(c) Six numbers are chosen from the integer numbers 5 to 14 (inclusive). **2**

Use the Pigeonhole Principle to prove that two of these digits must add to 19.

(d) The polynomial $P(x) = x^3 - kx^2 + (k-1)x - (k+2)$ has zeroes α, β and γ. **5**

If $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} \geq 2$, find the polynomial $P(x)$ with the smallest value of k.

End of Examination