Pymble Ladies' College

YEAR 11

YEARLY EXAMINATION

2021

Mathematics Extension 1

Section I

Multiple Choice Questions
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Instructions

- Answer using the Multiple Choice Answer Sheet provided.
- Use pencil for Questions 1-10

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SECTION I

10 Marks

Answer by using the Multiple Choice Answer Sheet.

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1 What is the inverse function of $f(x) = 3^x$?

(A) $f^{-1}(x) = x^3$

(B) $f^{-1}(x) = \log_3 x$

(C) $f^{-1}(x) = \ln x$

(D) $f^{-1}(x) = \log_x 3$

2 What kind of function/relation do the parametric equations $x = 3t, y = \frac{3}{t}$ represent?

(A) Line

(B) Circle

(C) Parabola

(D) Hyperbola

3 What is the solution of the inequality $x^2 - 4x - 12 \le 0$?

(A) $x \ge 12$ or $x \le -12$

(B) $-12 \le x \le 12$

(C) $x \ge 6$ or $x \le -2$

(D) $-2 \le x \le 6$

Please turn the page

4 What are the values of a and b if $\left(a-b\sqrt{2}\right)^2 = 59+30\sqrt{2}$ where a and b are both rational?

(A) $a=3, b=-5$

(B) $a=3, b=5$

(C) $a=5, b=3$

(D) $a=5, b=-3$

5 What is the solution of the inequality $\left|2x-4\right| > 6$?

(A) $-1 < x < 5$

(B) $x < -1$ or $x > 5$

(C) $-6 < x < 6$

(D) $x < -6$ or $x > 6$

6 Given the population of frogs in a small town at time t is $P = 2000e^{5t}$.
What is the instantaneous rate of change of the population when $t = 4$?

(A) $2000e^{20}$

(B) $5000e^{20}$

(C) $10000e^{20}$

(D) $400e^{20}$

Please turn the page

7 What is the value of $\cos^{-1}\cos\frac{7\pi}{6}$?

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{6}$

(D) $-\frac{\pi}{6}$

8 Which of the following is equivalent to $\cos 4\theta$?

(A) $\cos^2\theta - \sin^2\theta$

(B) $2\sin^2 2\theta - 1$

(C) $\cos^2 2\theta + \sin^2 2\theta$

(D) $2\cos^2 2\theta - 1$

9 What are the domain and range of the function $y = 2\cos^{-1}(3x-1)$?

(A) Domain: $-1 \le x \le 1$, Range: $0 \le y \le \pi$.

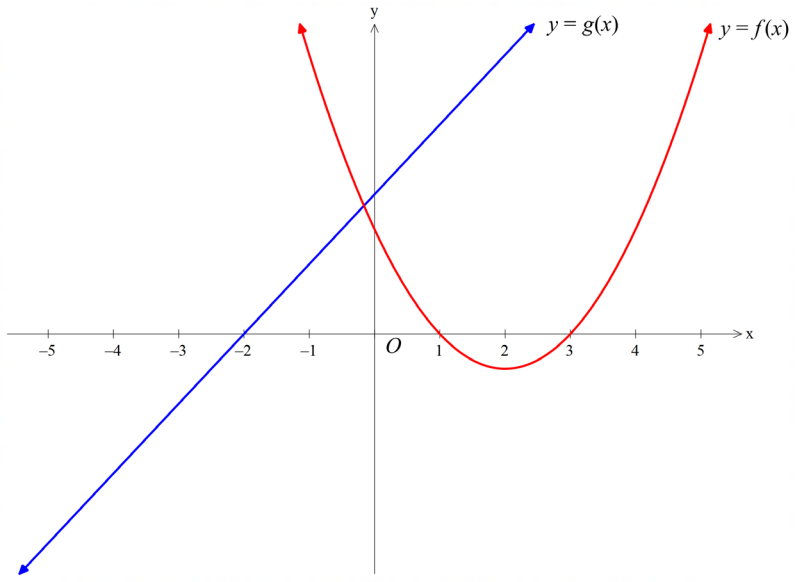
(B) Domain: $-\frac{2}{3} \le x \le \frac{2}{3}$, Range: $0 \le y \le 2\pi$.

(C) Domain: $0 \le x \le \frac{2}{3}$, Range: $0 \le y \le 2\pi$.

(D) Domain: $-\frac{2}{3} \le x \le \frac{2}{3}$, Range: $0 \le y \le \pi$.

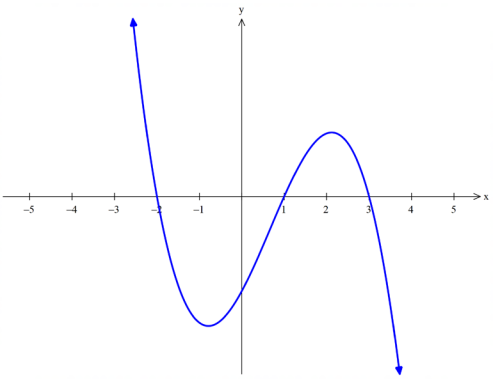
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10 Given the graphs of the functions $y = f(x)$ and $y = g(x)$ below.

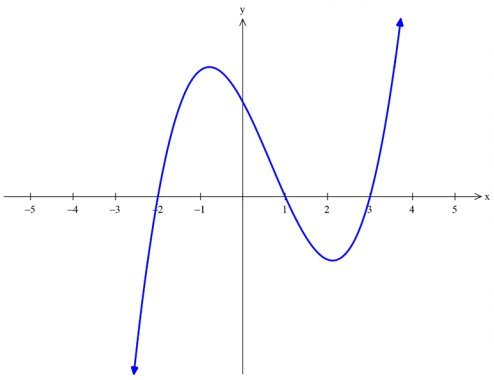


Which of the following graphs best represents the function $y = f(x)g(x)$?

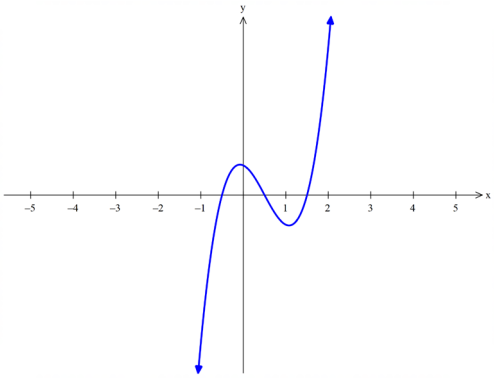
(A)



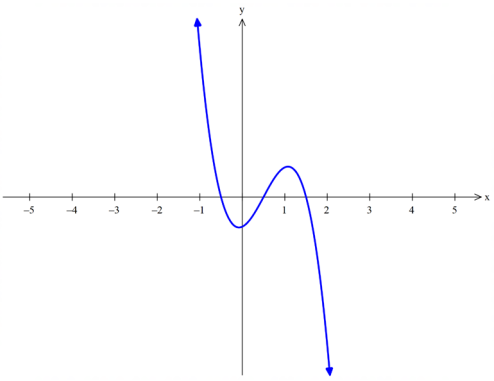
(B)



(C)



(D)



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Pymble Ladies' College

YEAR 11
YEARLY EXAMINATION
2021

Mathematics Extension 1
Section II
Question Booklet

50 marks
Attempt Questions 11-23
Allow about 1 hour and 45 minutes for this section

Instructions

- Answer these questions in the Answer Booklet provided.
- Answer in the spaces allocated for each question as these spaces provide guidance for the expected length of your response.
- Write using black or blue pen. Black is preferred.
- Erasable pens are **not** to be used.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of each part in the Answer booklet. If you use this space, clearly indicate which question you are answering.

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SECTION II

50 marks
Attempt all questions
Answer these questions in the Answer Book provided

Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space will be provided at the end of each part in the Answer Booklet.

PART A (10 marks)

Question 11 **Marks**

Find the cartesian equation of $x = 3\cos\theta - 1,\ y = 3\sin\theta + 2$ and describe it geometrically. **3**

Question 12 **Marks**

Find the exact value of $\tan 75°$. **2**

Question 13 **Marks**

The population of a colony of rabbits increases according to the law of exponential growth where $\frac{dP}{dt} = kP$, t is the time in months.

(a) Show that $P = Ae^{kt}$ satisfies the differential equation. **1**

(b) The initial population of the colony is 400 and it grows to 520 in three months. Find the values of A and k. **2**

(c) Find when the population first exceeds 4000. Give your answer correct to the nearest month. **2**

PART B (10 marks)

Question 14 **Marks**

Differentiate the following functions:

(a) $y = (3x-1)e^{-4x}$ **2**

(b) $y = \dfrac{2x^2-3}{e^{3x}}$ **2**

Question 15 **Marks**

Prove that $\dfrac{1+\cos 2\theta}{1-\cos 2\theta} = \cot^2\theta$. **2**

Question 16 **Marks**

Find the exact values of:

(a) $\sin\left(\cos^{-1}\left(-\dfrac{7}{12}\right)\right)$ **2**

(b) $\tan\left(2\tan^{-1}(-3)\right)$ **2**

PART C (10 marks)

Question 17 **Marks**

Solve $\frac{2x}{x-3} \leq 4$ **3**

Question 18 **Marks**

Consider the function $f(x) = \sqrt{2x+8}$

(a) State the domain of $f(x)$. **1**

(b) Find an expression of the inverse function $f^{-1}(x)$, clearly indicating the domain of $f^{-1}(x)$. **2**

(c) Find the point(s) of intersection of $y = f(x)$ and $y = f^{-1}(x)$. **2**

(d) On the set of axes that is provided in the answer booklet, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing all the relevant information. **2**

PART D (10 marks)

Question 19 **Marks**

(a) Show that $\sin\left(\sin^{-1}\frac{1}{\sqrt{5}}+\tan^{-1}\frac{1}{3}\right)=\frac{1}{\sqrt{2}}$ **3**

(b) Hence, find the exact value of $\sin^{-1}\frac{1}{\sqrt{5}}+\tan^{-1}\frac{1}{3}$ **1**

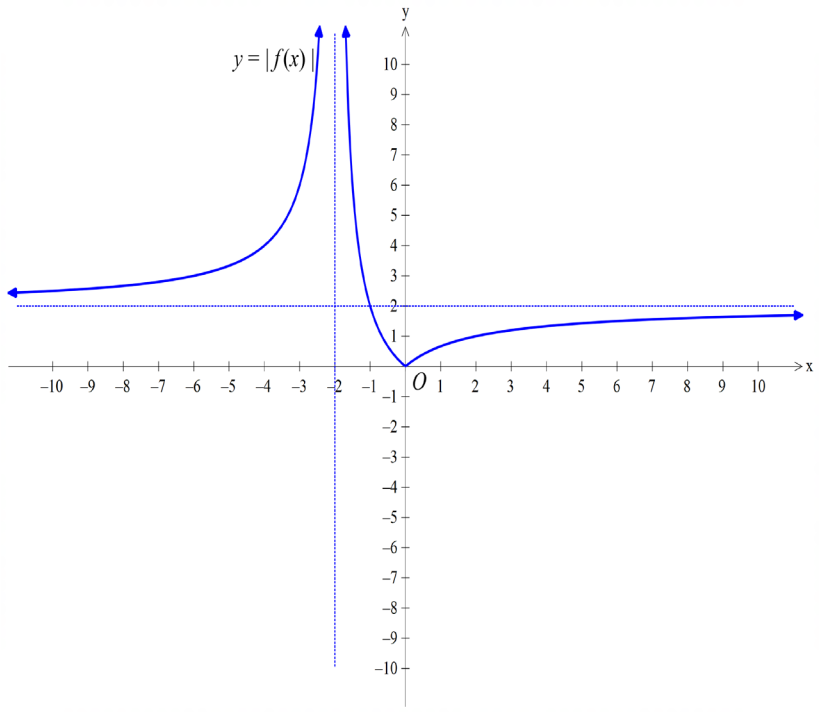
Question 20 **Marks**

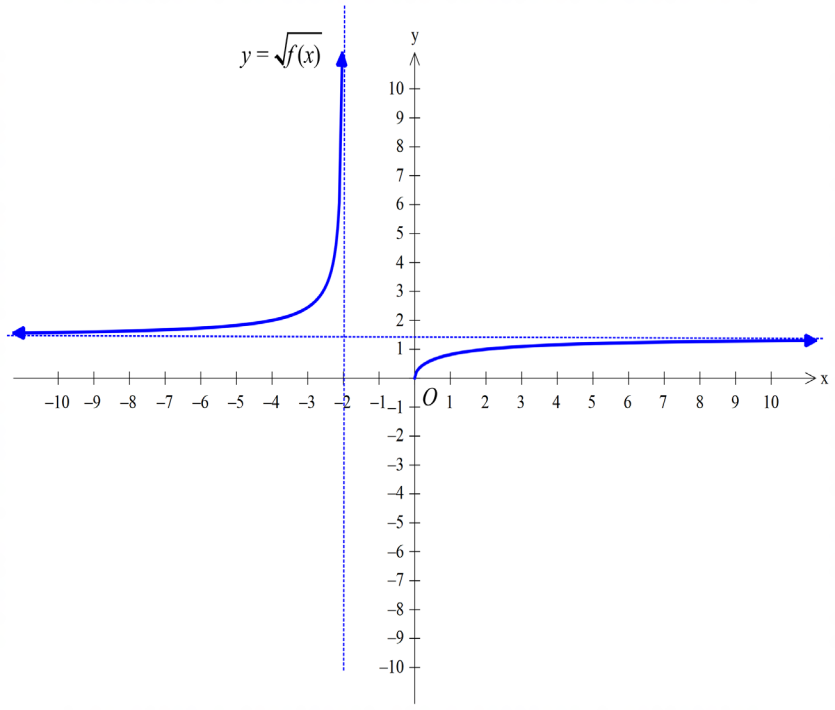
Find the values of m such that the line $y=mx-3$ touches the parabola $y=5x^2+2x+2$ at exactly one point. **2**

Question 21 on next page

Question 21 **Marks**

The graphs of $y = \left| f\left(x\right) \right|$ and $y = \sqrt{f\left(x\right)}$ are given below.





Sketch the following functions on the axes provided in the answer booklet.

(a) $y = f\left(x\right)$. **2**

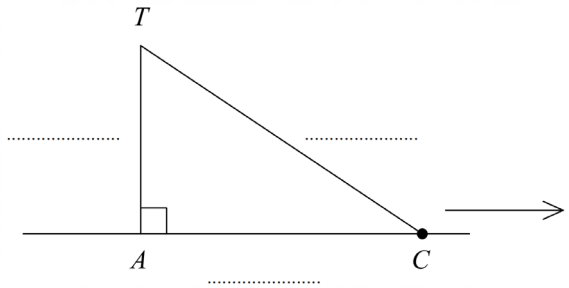
(b) $y = \dfrac{1}{f\left(x\right)}$ **2**

PART E (10 marks)

Question 22 **Marks**

A transmitter T is located 8 km from a straight road. The closest point on the road to the transmitter is A. A car (C) is travelling away from A along the road at a constant speed of 80 km/h. The distance from the car to A is x km and from the car to T is r km.

(a) Fill in the missing information on the diagram and find an expression of r in terms of x. The diagram is also provided in the Answer Booklet. **2**



(b) Find the rate at which the distance from the car to the transmitter is changing when the car is 6 km from A. **3**

Question 23 **Marks**

(a) Prove the identity $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ using the compound angle formulae. **3**

(b) Hence, find expressions for the exact values of the solutions to the equation $\dfrac{3x - x^3}{1 - 3x^2} = \sqrt{3}$. **2**

End of Paper