HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 11
Assessment 3 Term 3 2020

STUDENT NUMBER: _____

General Instructions

- Reading Time – 5 minutes
- Working Time – 80 minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators and drawing templates may be used
- In Questions 6 – 8, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 47

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

Section II Pages 4 – 7

42 marks

Attempt Questions 6 – 8.

Question	***1-5***	***6***	***7***	***8***	***Total***
Total	*/5*	*/14*	*/14*	*/14*	*/47*

This assessment task constitutes 40% of the Preliminary Higher School Certificate Course School Assessment

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

Q1. $sin(A+3B)=$

(A) $sin\,A\,cos\,3B - cos\,A\,sin\,3B$

(B) $sin\,3B\,cos\,A + cos\,A\,sin\,3B$

(C) $sin\,A\,cos\,3B + cos\,A\,sin\,3B$

(D) $sin\,A\,cos\,3B + cos\,3B\,sin\,A$

Q2. Which of the following is correct?

(A) $\sin^{-1}(-x) = -\sin^{-1}(x)$

(B) $\cos^{-1}(-x) = \pi + \cos^{-1}(x)$

(C) $\sin^{-1}(-x) = \sin^{-1}(x)$

(D) $\tan^{-1}(-x) = \tan^{-1}(x)$

Q3. What is the solution of $5^x = 4$?

(A) $x = \dfrac{\ln 4}{5}$

(B) $x = \dfrac{4}{\ln 5}$

(C) $x = \dfrac{\ln 4}{\ln 5}$

(D) $x = \ln\left(\dfrac{4}{5}\right)$

Q4. The derivative of $y = \dfrac{e^{2x}}{2x+1}$ is:

(A) $y' = \dfrac{xe^{2x}}{(2x+1)^2}$

(B) $y' = \dfrac{4xe^{2x}}{(2x+1)^2}$

(C) $y' = 4e^{2x}(x+1)$

(D) $y' = 2e^{2x}(x+1)$

Q5. If the parametric equations of a curve are $x = t - 1$ and $y = 2t + 1$, then the curve is a

(A) straight line

(B) circle

(C) parabola

(D) hyperbola

End of Section I

Section II

42 marks
Attempt Questions 6-8
Allow about 70 minutes for this section

Question 6 (14 Marks)

(a) Find the exact value of $\sin\left(\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)\right)$. **2**

(b) Make the *r* subject of the formula $A = P\left(1+\frac{r}{100}\right)^n$ **2**

(c) Sketch $y = 4\sin^{-1}(1-2x)$ showing all important features. **2**

(d) If α, β and γ are the roots of the cubic equation $5x^3 + 2x^2 - x - 3 = 0$, find the value of:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(e) For the polynomial $P(x) = x^3 - 7x^2 + 16x - 12$,

(i) Find $P'(x)$ **1**

(ii) Solve $P(x) = 0$, given that the polynomial has a root of multiplicity 2 (i.e. double root). **3**

Question 7 (14 marks) **START A NEW BOOKLET**

(a) Solve for x: $\dfrac{1}{x^2-2x} > \dfrac{1}{x^2-4}$ **3**

(b) (i) On the same number plane, draw a half page sketch of: **2**

$$y = |x+3| \text{ and } y = |x-1|$$

(ii) By addition of ordinates or otherwise, on the same number plane as (i), sketch: **2**

$$y = |x+3| + |x-1|$$

(iii) Hence, or otherwise solve: $|x+3| + |x-1| > 4$ **2**

Question 7 continues on next page.

Question 7 continued

(c) Newton's law of cooling states that for an object placed in surroundings at a constant temperature, the rate of change of the temperature of the object is proportional to the difference between the temperature of the object at any time, T, and its surroundings, A.

$$\text{i.e. } \frac{dT}{dt} = -k(T - A)$$

(i) Show that $T = A + Be^{-kt}$ is a solution to the equation $\frac{dT}{dt} = -k(T - A)$. **1**

(ii) The room in which the liquid has been placed has a constant temperature of $8^{\circ}C$. A liquid drops in temperature from $80^{\circ}C$ to $55^{\circ}C$ in 45 minutes. What are the values of A, B and k? Express k in exact form. **3**

(iii) How long will it take the liquid to reach a temperature of $40^{\circ}C$? Answer correct to the nearest minute. **1**

Question 8 (14 marks) **START A NEW BOOKLET**

(a) Show that the function $y = x^3 - 2x^2 + 11x - 8$ is increasing for all values of x. **2**

(b) Show that $\dfrac{\tan\left(\frac{\pi}{4}+\theta\right)}{\tan\left(\frac{\pi}{4}-\theta\right)} = \left(\dfrac{1+\tan\theta}{1-\tan\theta}\right)^2$ **2**

(c) (i) What is the radius of a sphere with surface area $1\,500\pi\ cm^2$? **1**

(ii) A spherical bubble is expanding so that its volume is increasing at a rate of $6\,cm^3/s$. Find the rate of increase of its radius when the surface area is $1\,500\pi\ cm^2$. Answer correct to 2 significant figures. **2**

(d) Bilbies and dingoes are Australian animals. A forest contains a number of bilbies. At a certain time, 8 dingoes are introduced into the forest and start eating the bilbies. As a consequence, the number of bilbies, *B*, decreases according to the formula:

$$B = 240 - e^{0.6t}$$

where t is the time in months after the dingoes are introduced.

(i) When will the population of bilbies be zero? Answer correct to the nearest month. **1**

(ii) Sketch the number of bilbies as a function of time. Show all important features. **1**

(iii) The number of dingoes is increasing according to the formula:

$$D = 8e^{0.3t}$$

When is the rate of increase of dingoes equal to the rate of decrease of bilbies? Answer correct to the nearest month. **2**

(iv) When is the number of dingoes equal to the number of bilbies? **3**

End of paper

Mathematics Extension 1

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Section I – Objective Response Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○