NAME MASTER

SYDNEY GRAMMAR SCHOOL

2018 Annual Examination

FORM V

MATHEMATICS EXTENSION 1

Monday 10th September 2018

General Instructions

- Writing time — 2 hours
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 87 Marks

- All questions may be attempted.

Section I — 9 Marks

- Questions 1—9 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II — 78 Marks

- Questions 10–15 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Ten.

5A: DNW	5B: DWH	5C: SDP
5D: LRP	5E: FMW	5F: NJL
5G: RR	5H: RDP	5I: LL

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 160 boys

Examiner
DNW

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What are the roots of the quadratic equation $x^2 + 2x - 4 = 0$?

(A) $1 - \sqrt{5}$ and $1 + \sqrt{5}$

(B) $-1 - \sqrt{5}$ and $-1 + \sqrt{5}$

(C) $1 - \sqrt{10}$ and $1 + \sqrt{10}$

(D) $-1 - \sqrt{10}$ and $-1 + \sqrt{10}$

QUESTION TWO

What is the equation of the vertical asymptote of the curve $y = \dfrac{x}{x-1}$?

(A) $y = -1$

(B) $y = 1$

(C) $x = -1$

(D) $x = 1$

QUESTION THREE

The quadratic equation $2x^2 + 3x - 4 = 0$ has roots α and β. What is the value of $\alpha\beta$?

(A) 2

(B) -2

(C) 4

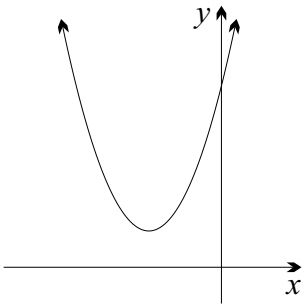
(D) -4

Examination continues next page ...

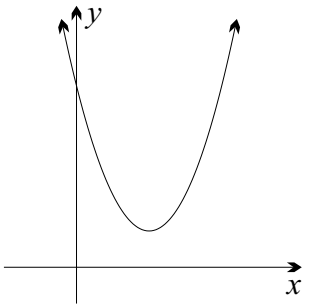
QUESTION FOUR

Which of the following graphs best represents the parabola with equation $y-1=(x+2)^2$?

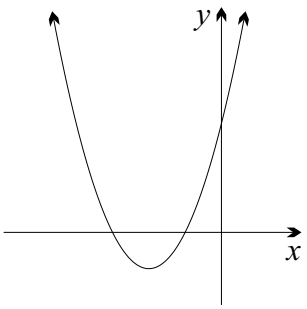
(A)



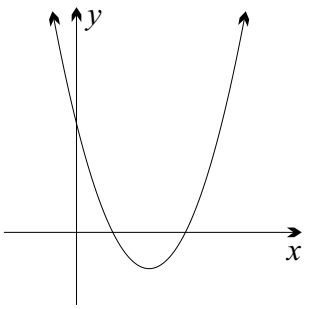
(B)



(C)



(D)



QUESTION FIVE

The integral $\displaystyle\int_0^3 |x-1|\,dx$ can be evaluated without finding a primitive.

What is its value?

(A) 5

(B) 3

(C) $2\frac{1}{2}$

(D) $1\frac{1}{2}$

QUESTION SIX

Suppose an integral is approximated with Simpson's Rule using nine function values. How many times is a parabola used to approximate the function?

(A) 9

(B) 8

(C) 4

(D) 3

Examination continues overleaf ...

QUESTION SEVEN

Which of the following is a primitive of $\dfrac{x}{x^2+4}$?

(A) $2\log(x^2+4)$

(B) $\log(x^2+4)$

(C) $\log(x+4)$

(D) $\frac{1}{2}\log(x^2+4)$

QUESTION EIGHT

What is the derivative of $\left(\sqrt{x}+1\right)^3$?

(A) $\dfrac{3}{\sqrt{x}}\left(\sqrt{x}+1\right)^2$

(B) $\dfrac{3}{2\sqrt{x}}\left(\sqrt{x}+1\right)^2$

(C) $\dfrac{3\sqrt{x}}{2}\left(\sqrt{x}+1\right)^2$

(D) $3\left(\sqrt{x}+1\right)^2$

QUESTION NINE

It is known that $f'(x) = (x-1)^2(x+1)$.

Which of the following features is the graph of $y = f(x)$ guaranteed to have?

(A) one turning point and one inflexion point

(B) one turning point and two inflexion points

(C) two turning points and one inflexion point

(D) two turning points and two inflexion points

End of Section I

Examination continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION TEN (13 marks) Use a separate writing booklet. **Marks**

(a) What is the gradient of the line $\dfrac{x}{4} - \dfrac{y}{6} = 1$? **1**

(b) Find the exact value of $\cos 135^\circ$. **1**

(c) Find rational numbers a and b such that $\dfrac{3}{5+\sqrt{7}} = a + b\sqrt{7}$. **2**

(d) Differentiate:

(i) $\sqrt{x+1}$ **1**

(ii) $e^{-x}\left(e^{-x} - 1\right)$ **2**

(iii) $\ln\left(x^3\right)$ **2**

(e) Determine:

(i) $\displaystyle\int \left(x^2 + 2x + 3\right)\, dx$ **1**

(ii) $\displaystyle\int \sqrt[3]{x}\, dx$ **1**

(iii) $\displaystyle\int \frac{1}{3x+2}\, dx$ **2**

Examination continues overleaf ...

QUESTION ELEVEN (13 marks) Use a separate writing booklet. **Marks**

(a) (i) How many multiples of 7 are there between 40 and 400? **1**

(ii) Find the sum of the multiples of 7 between 40 and 400. **2**

(b) Determine the monic quadratic equation with roots $3+\sqrt{2}$ and $3-\sqrt{2}$. Give your answer in the form $ax^2+bx+c=0$. **3**

(c) Let $A=(-1,3)$ and $B=(4,1)$. Let P be the point which divides AB internally in the ratio $\lambda : 1-\lambda$.

(i) Find the coordinates of the point P in terms of λ. **2**

(ii) Now suppose that P also lies on the line $2y=x+1$. Find the value of λ. **2**

(iii) Hence, or otherwise, find the point of intersection of the lines AB and $2y=x+1$. **1**

(d) Use the logarithm laws to help find y' for the function $y=\ln\left((x+3)\left(x-\frac{1}{2}\right)\right)$. **2**

Examination continues next page ...

QUESTION TWELVE (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the integral $\int_1^3 (x - \log_e x)\, dx$.

(i) Approximate this integral using the trapezoidal rule with three function values. Give your answer correct to one decimal place. **3**

(ii) Does the answer to part (i) underestimate or overestimate the exact value of the integral? Use calculus to justify your answer. **2**

(b) The quadratic equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots.

(i) Find the possible values of k. **2**

(ii) Determine the roots for each value of k found in part (i). **2**

(c) A curve with first derivative $\dfrac{dy}{dx} = 12x + 6$ passes through $(1, 2)$. Find its equation. **2**

(d) Show that $\dfrac{\cos\theta}{\tan\theta + \sec\theta} = 1 - \sin\theta$. **2**

Examination continues overleaf ...

QUESTION THIRTEEN (13 marks) Use a separate writing booklet. **Marks**

Consider the curve with equation $y = \dfrac{x^2 + x - 2}{x^2 - 4x + 4}$.

(a) Write the numerator and denominator in factored form. **1**

(b) Find the intercepts with the axes. **1**

(c) Determine whether the function is even, odd or neither. **1**

(d) Write down the equation of any vertical asymptote. **1**

(e) Show that $y = 1$ is the equation of the horizontal asymptote. **1**

(f) Find the coordinates of the point where the curve crosses the horizontal asymptote. **1**

(g) Show that there is a stationary point at $\left(\frac{2}{5}, -\frac{9}{16}\right)$ and determine its nature. **4**

(h) Sketch the curve showing all the features found in the previous parts. **2**

(i) State the range of the function $y = \dfrac{x^2 + x - 2}{x^2 - 4x + 4}$. **1**

Examination continues next page ...

QUESTION FOURTEEN (13 marks) Use a separate writing booklet. **Marks**

(a) (i) Determine $\dfrac{d}{dx}\left(x^3 \ln x\right)$. **2**

(ii) Hence evaluate $\displaystyle\int_1^e 3x^2 \ln x \, dx$. **3**

(b) (i) Find the coordinates of the two stationary points on the curve $y = x^3 e^{-x}$. **2**

(ii) Classify these stationary points. **3**

(c) Consider the function $f(x)$ defined by **3**

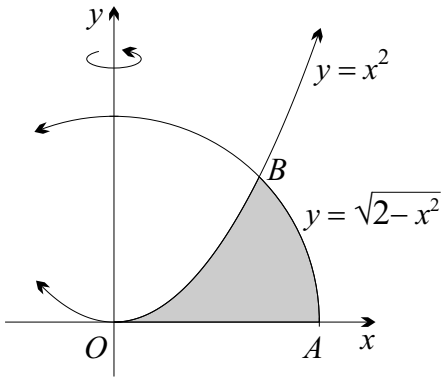
$$f(x) = \begin{cases} x^3 + x\,, & \text{for } x \leq 1 \\ e^{(x-1)} + 1\,, & \text{for } x > 1 \end{cases}$$

Show that $f(x)$ is continuous at $x = 1$.

Examination continues overleaf ...

QUESTION FIFTEEN (13 marks) Use a separate writing booklet. **Marks**

(a)



The diagram above shows the shaded region OAB in the first quadrant bounded by the parabola $y = x^2$, the semi-circle $y = \sqrt{2 - x^2}$ and the x-axis. This region is rotated about the y-axis to generate a solid of revolution.

(i) Find the y-coordinate of B. **1**

(ii) Write the volume of the solid as an integral. **2**

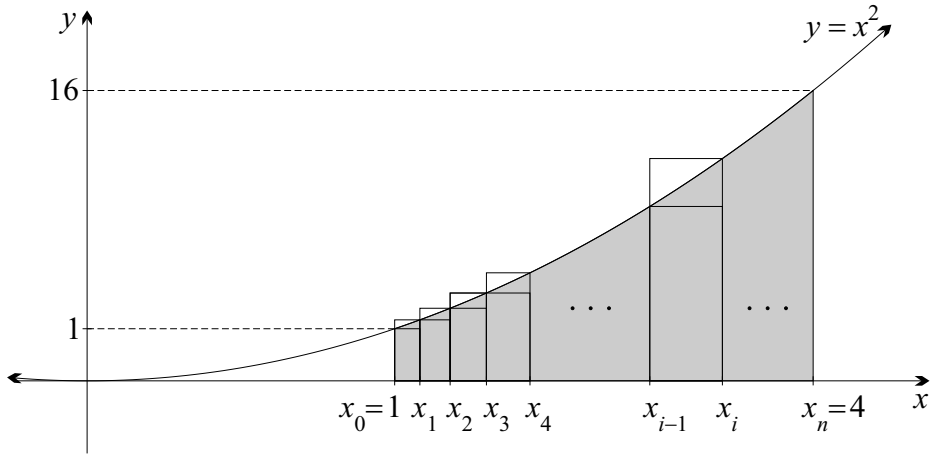
(iii) Hence find the volume of the solid. **1**

(b) Use the difference of two cubes identity to write $x^{3a} - 1$ as the product of two factors. **1**

Examination continues next page ...

QUESTION FIFTEEN (Continued)

(c)



The diagram above shows the shaded region under $y = x^2$ between $x = 1$ and $x = 4$. Also shown are upper- and lower-rectangles which will be used to find the exact area R of this region. In each case, there are n rectangles of unequal width.

Let A_i be the area of the ith upper-rectangle. The x-coordinate at the bottom right corner of this rectangle is $x_i = 4^{i/n}$. That is, $x_i = \left(\sqrt[n]{4}\right)^i$.

(i) Show that $A_i = 4^{(3i-1)/n} \times \left(4^{1/n} - 1\right)$. **2**

(ii) Hence show that: **3**

$$R \leq \frac{4^{2/n} \times \left(4^{1/n} - 1\right) \times 63}{4^{3/n} - 1}$$

(iii) Let a_i be the area of the ith lower-rectangle. Show that $a_i = A_i \times 4^{-2/n}$. **1**

(iv) It follows that: **2**

$$\frac{\left(4^{1/n} - 1\right) \times 63}{4^{3/n} - 1} \leq R \leq \frac{4^{2/n} \times \left(4^{1/n} - 1\right) \times 63}{4^{3/n} - 1}.$$

Do NOT prove this result.

Use this result to determine the value of R. You may assume that $\lim\limits_{n\to\infty} 4^{1/n} = 1$.

End of Section II

END OF EXAMINATION