Name: _____

KNOX GRAMMAR SCHOOL **Teacher:** _____

2017

Year 11 Yearly Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen only
- The official NESA Reference sheet is provided
- Board-approved calculators may be used
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Teachers:
Mr Vuletich
Mr Sedgman
Mr Mulray
Mr Willcocks (Examiner)

This paper MUST NOT be removed from the examination room

Total marks – 70

Section I: Pages 3 – 5

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II: Pages 6 – 11

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Write your full name and your teacher's name on the front cover of each writing booklet

Number of Students in Course: 96

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 If $x+\frac{1}{x}=2$, what is the value of $x^2+\frac{1}{x^2}$?

(A) 2 (B) 4 (C) 6 (D) 8

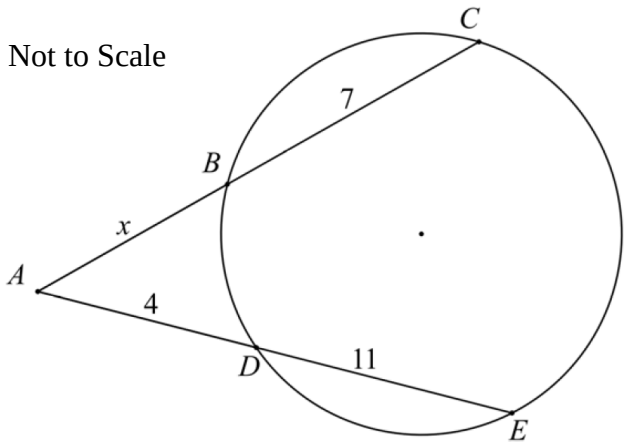
2 What is the natural domain of the function $y=\sqrt{x+1}-\sqrt{x+2}$?

(A) $x \le -2$ (B) $x \ge -1$

(C) $-2 \le x \le -1$ (D) $x \le -2$ or $x \ge -1$

3 In the diagram below, *BC* and *DE* are chords of a circle. *CB* and *ED* are produced to meet at *A*. What is the value of *x*?

Not to Scale



(A) $\frac{28}{11}$ (B) $\frac{11}{28}$ (C) 5 (D) 12

4 What is the inverse function of $f(x)=x^2+1$ for $x \le 0$?

(A) $f^{-1}(x)=-\sqrt{x-1}$, for $x \le 0$ (B) $f^{-1}(x)=\sqrt{x-1}$, for $x \le 0$

(C) $f^{-1}(x)=-\sqrt{x-1}$, for $x \ge 1$ (D) $f^{-1}(x)=\sqrt{x-1}$, for $x \ge 1$

Section I continued on the following page

5 The angle between the lines $2x+y=4$ and $x+y=2$, to the nearest degree, is:

(A) 18° (B) 25° (C) 45° (D) 72°

6 A parabola has parametric coordinates $\left(6t,-3t^2\right)$. The coordinates of the focus are:

(A) $\left(0,\frac{1}{4}\right)$ (B) $\left(0,-\frac{1}{4}\right)$ (C) $(0,3)$ (D) $(0,-3)$

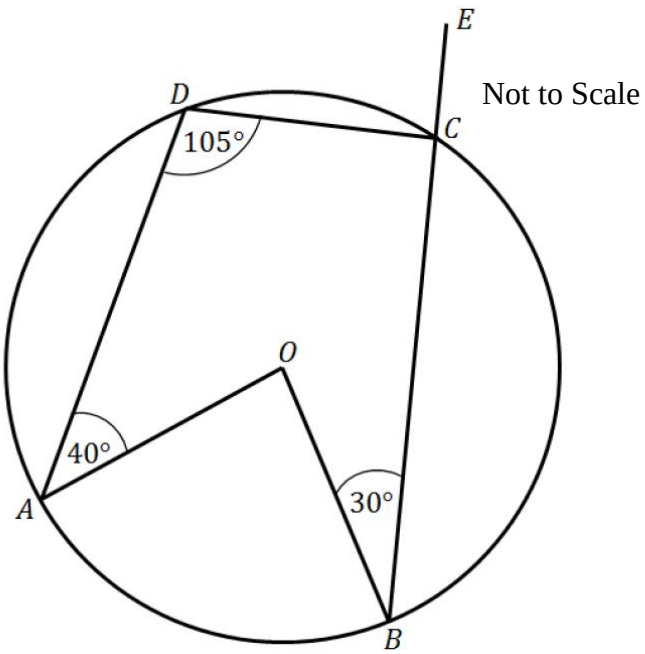
7 At a certain instant, a sphere is of radius 10 cm and the radius is increasing at a rate of 2 cm/s. The rate of increase of the volume of the sphere at that instant, in cm^3/s is:

(A) 80π (B) $\frac{800\pi}{3}$ (C) 800π (D) 400π

8 The domain of $f(x)=\log_e\left[(x-4)(5-x)\right]$ is:

(A) $4<x<5$ (B) $x<4,\ x>5$ (C) $4\le x\le 5$ (D) $x\le 4,\ x\ge 5$

9 In the diagram below, O is the centre of the circle $ABCD$ and BCE is a straight line. If $\angle ADC=105°$, $\angle OBC=30°$ and $\angle OAD=40°$, find the size of $\angle DCE$.

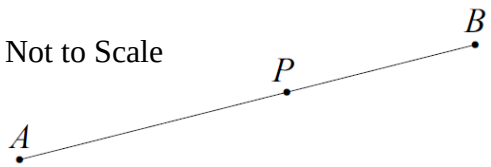


Not to Scale

(A) 80° (B) 85° (C) 90° (D) 95°

Section I continued on the following page

10 In the diagram below the point P divides the interval AB in the ratio $3:2$. In what ratio does the point A divide the interval BP?



(A) $-5:3$ (B) $-5:2$ (C) $-3:5$ (D) $-2:5$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) Write $\sqrt{\frac{3^{5k+2}}{27^{k}}}$ in simplest index form. **2**

(b) Sketch the parabola $y^2+8x-2y+25=0$, clearly showing the location of the vertex, focus and directrix. **3**

(c) The point $(6,k)$ is 8 units from the straight line $3x+4y+2=0$. Find the value(s) of k. **2**

(d) The points $P(2ap,ap^2)$ and $Q(2aq,aq^2)$ lie on the parabola $x^2=4ay$. If the tangents at P and Q intersect at $45°$ show that $|1+pq|=|p-q|$. **2**

(e) Given that α and $m\alpha$ are the roots of the equation $x^2+px+q=0$, show that $mp^2=(m+1)^2 q$. **2**

(f) By eliminating θ from $x=5\sin\theta$ and $y=5\cos\theta+1$, find the Cartesian equation of the locus of the points relating x and y. **2**

(g) Show that the series $\log_2 x+\log_4 x+\log_{16} x+...$ is geometric and find the sum of the series for infinite terms. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

Marks

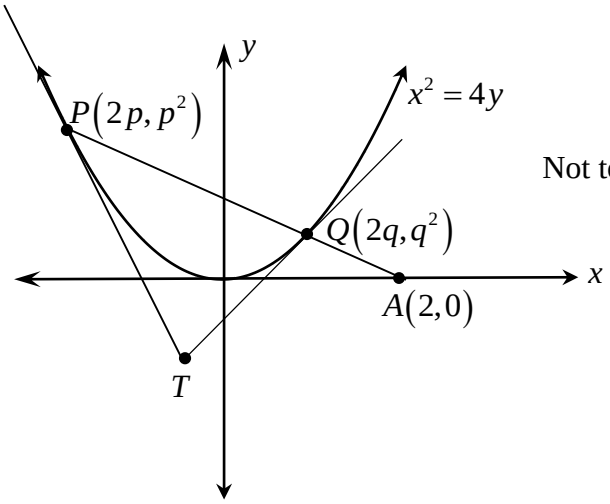
(a) Consider the function $f(x)=\dfrac{2x}{x^2-9}$.

(i) Show that $f(x)$ is odd. **1**

(ii) By first finding $\lim\limits_{x\to\infty}\dfrac{2x}{x^2-9}$, give the equation of any horizontal asymptotes. **2**

(iii) Without the use of Calculus and using about half a page, sketch a neat graph of $f(x)$, clearly labelling all intercepts and asymptotes. **2**

(b) Let $P(2p,p^2)$ and $Q(2q,q^2)$ be two points on the parabola $x^2=4y$. The tangents to the parabola at *P* and *Q* intersect at *T*. The secant *PQ* passes through the point $A(2,0)$.



Not to Scale

(i) Show that $p+q=pq$. **3**

(ii) Assuming that the equations of the tangents at *P* and *Q* are $y=px-p^2$ and $y=qx-q^2$ respectively, determine the Cartesian equation of the locus of *T*. **2**

(c) (i) Sketch the graph of $g(x)=\left|x^2-2x-8\right|$, labelling the *x* and *y* intercepts. **2**

(ii) Hence or otherwise, find the **number** of solutions to $g(x)=9$. **1**

(iii) Show that $g(x)$ is continuous but not differentiable at $x=4$. **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

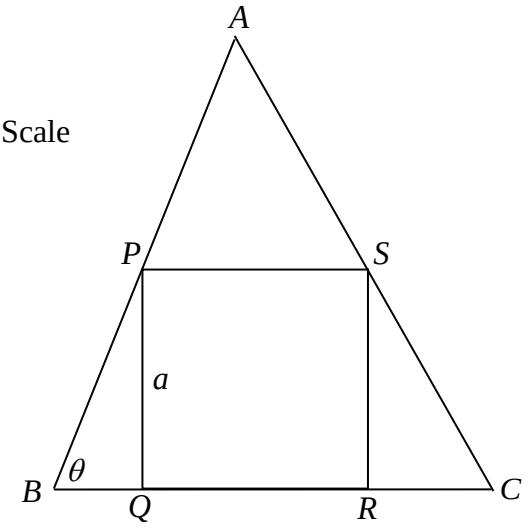
Marks

(a) Solve the inequation $\frac{x}{2x+1} \leq 2$. **3**

(b) Find $\frac{d}{dx}\left(\frac{2x-1}{\sqrt{2x-1}}\right)$. **2**

(c) $PQRS$ is a square inscribed in the isosceles ΔABC with $AB = AC$ and $PQ = a$.

Not to Scale



Find the length of AB in terms of a, $\sin\theta$ and $\cos\theta$. **2**

(d) The point R divides the interval joining $P(a, 2b)$ and $Q(3a, -b)$ externally in the ratio $2:3$. What are the coordinates of R? **2**

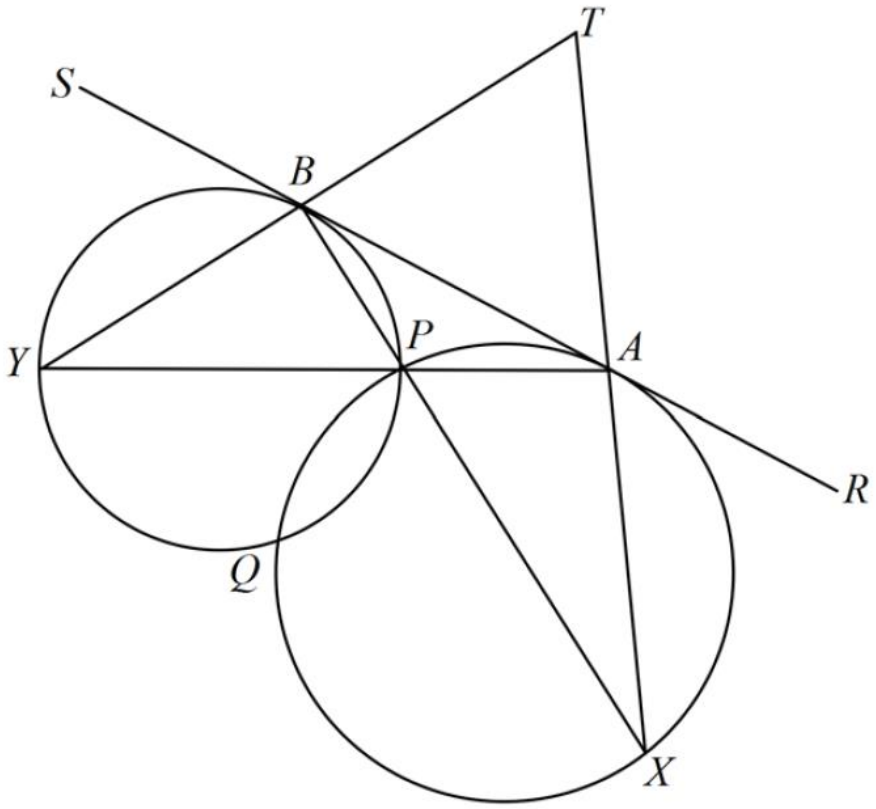
(e) Show that the sum of the series: **2**

$\log_{10}(x-2) + \log_{10}(x-2)^2 + \log_{10}(x-2)^3 + \cdots + \log_{10}(x-2)^n$ is equal to $\frac{n}{2}\log_{10}(x-2)^{n+1}$.

Question 13 continued on the following page

Marks

(f)



In the diagram above, the line SR is tangent to circles $AXQP$ and $BPQY$ at A and B respectively. The chords YB and XA when produced, meet at T. The points A, P and Y are collinear and similarly for the points B, P and X.

Refer to your separate answer booklet for this question.

(i) Explain why $\angle SBY = \angle BPY$. **1**

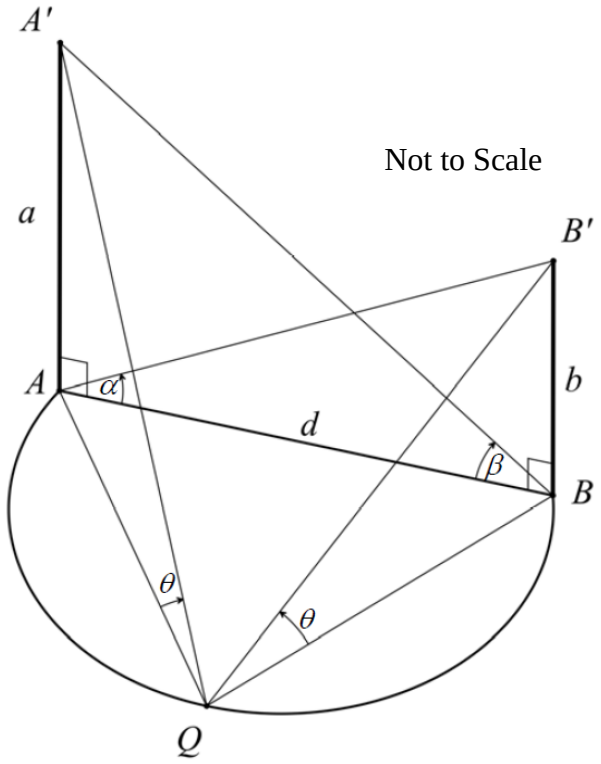
(ii) Prove that $AT = TB$. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

Marks

(a) AQB is a semi-circle in the horizontal plane with diameter AB of length d metres. There are two vertical posts AA' and BB' of heights a and b respectively. From Q, the angle of elevation to the tops of both posts A' and B' is θ. From A the angle of elevation to B' is α and from B the angle of elevation to A' is β.



(i) Prove that $d^2 = \dfrac{a^2 + b^2}{\tan^2 \theta}$. **3**

(ii) Show that $\tan^2 \alpha + \tan^2 \beta = \tan^2 \theta$. **2**

(b) Mason makes a deposit of \$5000 at the start of each year into a savings account. He earns monthly compound interest on his savings account at 4·8% per annum. Let A_n be the value of the account at the end of n years.

(i) Show that $A_1 = \$5245.35$. **1**

(ii) Show that $A_2 = \$5000\left(1.004^{12} + 1.004^{24}\right)$. **1**

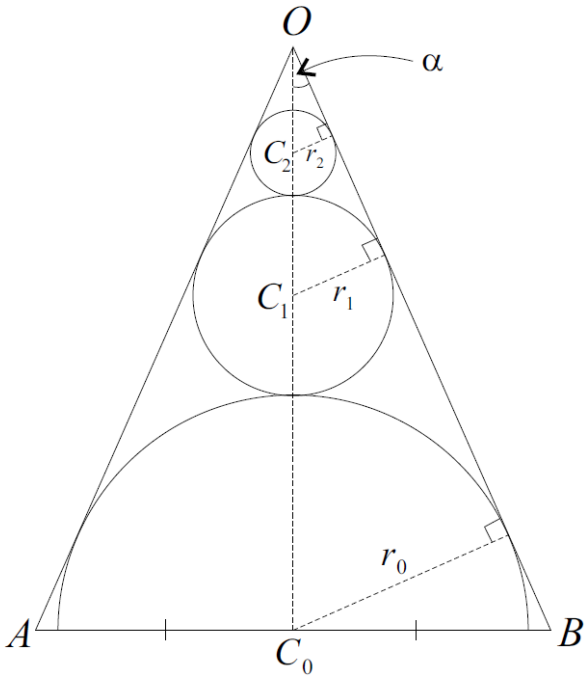
(iii) Show that $A_n = \dfrac{\$5000 \times 1.004^{12} \times \left(1.004^{12n} - 1\right)}{1.004^{12} - 1}$. **2**

(iv) By the end of which year will Mason have over \$100 000 in his account? **2**

Question 14 continued on the following page

Marks

(c) In the diagram below ΔOAB is isosceles, with $OA = OB$ and $\angle AOB = 2\alpha$. The semicircle of radius r_0 has centre C_0, the midpoint of AB and the sides OA and OB are tangents. Infinitely many circles are stacked on top of the semicircle so that each circle is tangent to the circles immediately above and below it, and also tangent to OA and OB. The circles have radii $r_1, r_2, r_3, \ldots$ and centres $C_1, C_2, C_3, \ldots$ The semicircle and the first two circles are shown in the diagram.



(i) Write down an infinite series for OC_0 in terms of $r_0, r_1, r_2, \ldots$ and write down a similar series for OC_1. **1**

(ii) Prove that $\dfrac{r_1}{r_0} = \dfrac{1-\sin\alpha}{1+\sin\alpha}$. **3**

End of Examination