Perms

Combs

Pigeonhole

Binomial

Probability

Question 1: ME1 HSC 2015 4 1

A rowing team consists of 8 rowers and a coxswain.

The rowers are selected from 12 students in Year 10.

The coxswain is selected from 4 students in Year 9.

In how many ways could the team be selected?

(A) ${}^{12}C_8 + {}^{4}C_1$

(B) ${}^{12}P_8 + {}^{4}P_1$

(C) ${}^{12}C_8 \times {}^{4}C_1$

(D) ${}^{12}P_8 \times {}^{4}P_1$

Question 2: ME1 HSC 2012 5 1

How many arrangements of the letters of the OLYMPIC are possible if the C and L are to be together in any order?

(A) 5!

(B) 6!

(C) 2×5!

(D) 2×6!

Question 3: ME1 HSC 2013 7 1

A family of eight is seated randomly around a circular table. What is the probability that the two youngest members of the family sit together.

(A) $\frac{6!2!}{7!}$

(B) $\frac{6!}{7!2!}$

(C) $\frac{6!2!}{8!}$

(D) $\frac{6!}{8!2!}$

Question 4: ME2 HSC 2015 7 1

The numbers $1, 2, \ldots n$, for $n \geq 4$, are randomly arranged in a row

What is the probability that the number 1 is somewhere to the left of the number 2?

(A) $\frac{1}{2}$

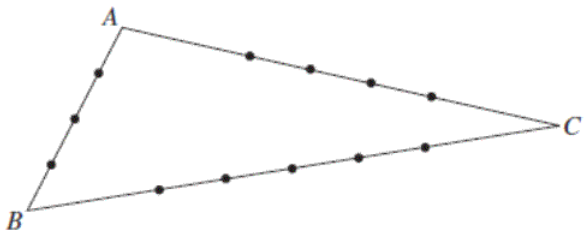
(B) $\frac{1}{n}$

(C) $\frac{1}{2(n-2)!}$

(D) $\frac{1}{2(n-1)!}$

Question 5: ME1 HSC 2022 7 1

The diagram shows triangle ABC with points chosen on each of the sides. On side AB, 3 points are chosen. On side AC, 4 points are chosen. On side BC, 5 points are chosen.



How many triangles can be formed using the chosen points as vertices?

A. 60

B. 145

C. 205

D. 220

Question 6: ME1 HSC 2014 8 1

In how many ways can 6 people from a group of 15 people be chosen and then arranged in a circle?

(A) $\frac{14!}{8!}$

(B) $\frac{14!}{8!6}$

(C) $\frac{15!}{9!}$

(D) $\frac{15!}{9!6}$

Question 7: ME1 HSC 2016 8 1

A team of 11 students is to be formed from a group of 18 students. Among the 18 students are 3 students who are left-handed.

What is the number of possible teams containing at least 1 student who is left-handed?

(A) 19 448
(B) 30 459
(C) 31 824
(D) 58 344

Question 8: ME1 HSC 2018 8 1

Six men and six women are to be seated at a round table.

In how many different ways can they be seated if men and women alternate?

(A) 5! 5!
(B) 5! 6!
(C) 2! 5! 5!
(D) 5! 5! 6!

Question 9: ME1 HSC 2019 8 1

In how many ways can all the letters of the word PARALLEL be placed in a line with the three Ls together?

(A) $\frac{6!}{2!}$
(B) $\frac{6!}{2!3!}$
(C) $\frac{8!}{2!}$
(D) $\frac{8!}{2!3!}$

Question 10: ME1 HSC 2020 8 1

Out of 10 contestants, six are to be selected for the final round of a competition. Four of those six will be placed 1st, 2nd, 3rd and 4th.

In how many ways can this process be carried out?

A. $\frac{10!}{6!4!}$

B. $\frac{10!}{6!}$

C. $\frac{10!}{4!2!}$

D. $\frac{10!}{4!4!}$

Question 11: ME2 HSC 2013 10 1

A hostel has four vacant rooms. Each room can accomodate a maximum of four people.

In how many different ways can six people be accomodated in the four rooms?

(A) 4020
(B) 4068
(C) 4080
(D) 4096

Question 12: ME1 HSC 2021 10 1

The members of a club voted for a new president. There were 15 candidates for the position of president and 3543 members voted. Each member voted for one candidate only.

One candidate received more votes than anyone else and so became the new president.

What is the smallest number of votes the new president could have received?

A. 236
B. 237
C. 238
D. 239

Question 13: ME1 HSC 2001 2 (c) (i) 1

How many arrangements of the letters in the word ALGEBRAIC are possible?

Question 14: ME1 HSC 2001 2 (c) (ii) **2**

How many arrangements of the letters in the word ALGEBRAIC are possible if the vowels must occupy the 2nd, 3rd, 5th and 8th position?

The letters A, E, I, O and U are vowels.

Question 15: ME1 HSC 2004 2 (e) (i) **1**

A four-person team is to be chosen at random from nine women and seven men.

In how many ways can this team be chosen?

Question 16: ME1 HSC 2011 2 (e) (i) **1**

Alex′s playlist consists of 40 different songs that can be arranged in any order.

How many arrangements are there for the 40 songs?

Question 17: ME1 HSC 2011 2 (e) (ii) **1**

Alex′s playlist consists of 40 different songs that can be arranged in any order.

From part (i), you have found that there are 40! arrangements for the 40 songs?

Alex decides that she wants to play her three favourite songs first, in any order.

How many arrangements of the 40 songs are now possible?

Question 18: ME1 HSC 2004 2 (e) (ii) **1**

A four-person team is to be chosen at random from nine women and seven men.

From part (i), you have found that this team be chosen in 1820 ways.

What is the probability that the team will consist of four women?

Question 19: ME1 HSC 2003 3 (a) **2**

How many nine-letter arrangements can be made using the letters of the word ISOSCELES?

Question 20: ME1 HSC 2002 3 (a) (i) **1**

Seven people are to be seated at a round table.

How many seating arrangements are possible?

Question 21: ME1 HSC 2010 3 (a) (i) 1

At the front of a building there are five garage doors. Two of the doors are to be painted red, one is to be painted green, one blue and one orange.

How many possible arrangements are there for the colours on the doors?

Question 22: ME1 HSC 2002 3 (a) (ii) 2

Seven people are to be seated at a round table.

From part (i), you have found that there are 6! possible arrangements.

Two people, Kevin and Jill, refuse to sit next to each other. How many seating arrangements are then possible?

Question 23: ME1 HSC 2010 3 (a) (ii) 1

At the front of a building there are five garage doors. Two of the doors are to be painted red, one is to be painted green, one blue and one orange.

From part (i), you have found that there are $\frac{5!}{2!}$ possible arrangements for the colours on the doors?

How many possible arrangements are there for the colours on the doors if the two red doors are next to each other?

Question 24: ME1 HSC 2006 3 (c) (i) 1

Sophie has five coloured blocks: one red, one blue, one green, one yellow and one white. She stacks two, three, four or five blocks on top of one another to form a vertical tower.

How many different towers are there that she could form that are three blocks high?

Question 25: ME1 HSC 2006 3 (c) (ii) 2

Sophie has five coloured blocks: one red, one blue, one green, one yellow and one white. She stacks two, three, four or five blocks on top of one another to form a vertical tower.

From part (i), you have shown that 60 different towers can be formed that are three blocks high.

How many different towers can she form in total?

Question 26: ME1 HSC 2003 4 (a) 1

A committee of 6 is to be chosen from 14 candidates. In how many different ways can this be done?

Question 27: ME1 HSC 2008 4 (b) (i) 1

Barbara and John and six other people go through a doorway one at a time.

In how many ways can the eight people go through the doorway if John goes through the doorway after Barbara with no-one in between?

Question 28: ME1 HSC 2008 4 (b) (ii) 1

Barbara and John and six other people go through a doorway one at a time.

Find the number of ways in which the eight people can go through the doorway if John goes through the doorway after Barbara.

Question 29: ME2 HSC 2003 4 (c) (i) 1

A hall has n doors. Suppose that n people each choose any door at random to enter the hall.

In how many ways can this be done?

Question 30: ME2 HSC 2002 4 (c) (i) 1

From a pack of nine cards numbered $1, 2, 3, \ldots, 9$, three cards are drawn at random and laid on a table from left to right.

What is the probability that the number formed exceeds 400?

Question 31: ME2 HSC 2002 4 (c) (ii) 2

From a pack of nine cards numbered $1, 2, 3, \ldots, 9$, three cards are drawn at random and laid on a table from left to right.

From part (i), you found that the probability that the number formed exceeds 400 is $\frac{2}{3}$.

What is the probability that the digits are drawn in descending order?

Question 32: ME2 HSC 2003 4 (c) (ii) 2

A hall has n doors. Suppose that n people each choose any door at random to enter the hall.

From part (i), you found that there are n^n ways for n people to choose any door at random to enter the hall.

What is the probability that at least one door will not be chosen by any of the people?

Question 33: ME2 HSC 2010 4 (d) (i) 1

A group of 21 people is to be divided into discussion groups.

In how many ways can the discussion groups be formed if there are 8 people in one group, and 4 people in another?

Question 34: ME2 HSC 2010 4 (d) (ii) 2

A group of 21 people is to be divided into discussion groups.

From part (i),

In how many ways can the discussion groups be formed if there are 3 groups containing 4 people each?

Question 35: ME1 HSC 2007 5 (b) 2

Mr and Mrs Roberts and their four children go to the theatre. They are randomly allocated six adjacent seats in a single row.

What is the probability that the four children are allocated seats next to each other?

Question 36: ME2 HSC 2005 5 (b) (i) 1

Mary and Ferdinand are competing against each other in a competition in which the winner is the first to score five goals. The outcome is recorded by listing, in order, the initial of the person who scores each goal. For example, one possible outcome would be recorded as $MFFMMFMM$.

Explain why there are five different ways in which the outcome could be recorded if Ferdinand scores only one goal in the competition.

Question 37: ME2 HSC 2004 5 (b) (ii) Similar to Q 11. 2

From part (i), you showed that the number of ways n students be place.d in two distinct rooms so that neither room is empty is $2^n - 2$

In how many ways can five students be placed in three distinct rooms so that no room is empty?

Question 38: ME2 HSC 2005 5 (b) (ii) 2

Mary and Ferdinand are competing against each other in a competition in which the winner is the first to score five goals. The outcome is recorded by listing, in order, the initial of the person who scores each goal. For example, one possible outcome would be recorded as $MFFMMFMM$.

From part (i), you found that there are five different ways in which the outcome could be recorded if Ferdinand scores only one goal in the competition.

In how many ways could the outcome of this competition be recorded?

Question 39: ME2 HSC 2001 5 (c) (i) 2

A class of 22 students is to be divided into four groups consisting of 4, 5, 6 and 7 students.

In how many ways can this be done? Leave your answer in unsimplified form.

Question 40: ME2 HSC 2001 5 (c) (ii) 2

A class of 22 students is to be divided into four groups consisting of 4, 5, 6 and 7 students.

From part (i), you have found that the number of ways to divide a class of 22 students into four groups consisting of 4,5,6 and 7 students is $\frac{22!}{4!5!6!7!}$.

Suppose that the four groups have been chosen.

In how many ways can the 22 students be arranged around a circular table if the students in each group are to be seated together? Leave your answer in unsimplified form.

Question 41: ME2 HSC 2006 5 (d) (i) 1

In a chess match between the Home team and the Away team, a game is played on each of board 1, board 2, board 3 and board 4.

On each board, the probability that the Home team wins is 0.2, the probability of a draw is 0.6 and the probability that the Home team loses is 0.2.

The results are recorded by listing the outcomes of the games for the Home team in board order. For example, if the Home team wins on board 1, draws on board 2, loses on board 3 and draws on board 4, the result is recorded as WDLD.

How many different recordings are possible?

Question 42: ME2 HSC 2006 5 (d) (ii) 1

In a chess match between the Home team and the Away team, a game is played on each of board 1, board 2, board 3 and board 4.

On each board, the probability that the Home team wins is 0.2, the probability of a draw is 0.6 and the probability that the Home team loses is 0.2.

The results are recorded by listing the outcomes of the games for the Home team in board order. For example, if the Home team wins on board 1, draws on board 2, loses on board 3 and draws on board 4, the result is recorded as WDLD.

From part (i), you found that there are 81 different recordings possible.

Calculate the probability of the result which is recorded as WDLD.

Question 43: ME2 HSC 2006 5 (d) (iii) 3

In a chess match between the Home team and the Away team, a game is played on each of board 1, board 2, board 3 and board 4.

On each board, the probability that the Home team wins is 0.2, the probability of a draw is 0.6 and the probability that the Home team loses is 0.2.

The results are recorded by listing the outcomes of the games for the Home team in board order. For example,if the Home team wins on board 1, draws on board 2, loses on board 3 and draws on board 4, the result is recorded as WDLD.

From part (i), you found that there are 81 different recordings possible.

From part (ii), you found that $P(\text{WDLD}) = \dfrac{9}{625}$.

Teams score 1 point for each game won, $\dfrac{1}{2}$ a point for each game drawn and 0 points for each game drawn and 0 points for each game lost.

What is the probability that the Home team scores more points than the Away team?

Question 44: ME2 HSC 2008 7 (a) (i) 2

An urn contains n red balls, n white balls and n blue balls. Three balls are drawn at random from the urn, one at a time, without replacement.

What is the probability, p_s, that the three balls are all the same colour?

Question 45: ME2 HSC 2008 7 (a) (ii) 1

An urn contains n red balls, n white balls and n blue balls. Three balls are drawn at random from the urn, one at a time, without replacement.

From part (i), you found that the probability, p_s, that the three balls are all the same colour is

$$p_s = \frac{(n-1)(n-2)}{(3n-1)(3n-2)}$$

What is the probability, p_d, that the three balls are all of different colours?

Question 46: ME2 HSC 2008 7 (a) (iii) 1

An urn contains n red balls, n white balls and n blue balls. Three balls are drawn at random from the urn, one at a time, without replacement.

From part (i), you found that the probability that the three balls are all the same colour is

$$p_s = \frac{(n-1)(n-2)}{(3n-1)(3n-2)}$$

From part (ii), you found that the probability that the three balls are all of different colours is

$$p_d = \frac{2n^2}{(3n-1)(3n-2)}$$

What is the probability, p_m, that two balls are of one colour and the third is of a different colour?

Question 47: ME2 HSC 2008 7 (a) (iv) 1

An urn contains n red balls, n white balls and n blue balls. Three balls are drawn at random from the urn, one at a time, without replacement.

From part (i), you found that the probability that the three balls are all the same colour is

$$p_s = \frac{(n-1)(n-2)}{(3n-1)(3n-2)}$$

From part (ii), you found that the probability that the three balls are all of different colours is

$$p_d = \frac{2n^2}{(3n-1)(3n-2)}$$

From part (iii), you found that the probability that two balls are of one colour and the third is of a different colour is

$$p_m = \frac{6n(n-1)}{(3n-1)(3n-2)}$$

If n is large, what is the approximate ratio $p_s : p_d : p_m$?

Question 48: ME1 HSC 2004 7 (b) (iii) 1

From part (i), you have shown that for all positive integers n,

$$x\left[(1+x)^{n-1}+(1+x)^{n-2}+\ldots+(1+x)^2+(1+x)+1\right]=(1+x)^n-1$$

From part (ii), you have shown that for $1\le k\le n$,

$$\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}+\ldots+\binom{k-1}{k-1}=\binom{n}{k}$$

Show that $n\binom{n-1}{k}=(k+1)\binom{n}{k+1}$.

Question 49: ME1 HSC 2010 7 (c) (i) 1

A box contains n identical red balls and n identical blue balls. A selection of r balls is made from the box, where $0\le r\le n$.

Explain why the number of possible colour combinations is $r+1$.

Question 50: ME1 HSC 2010 7 (c) (ii) 1

A box contains n identical red balls and n identical blue balls. A selection of r balls is made from the box, where $0\le r\le n$.

Explain why the number of possible colour combinations is $r+1$.

Another box contains n white balls labelled consecutively from 1 to n. A selection of $n-r$ balls is made from the box, where $0\le r\le n$.

Explain why the number of different selections is $\binom{n}{r}$.

Question 51: ME2 HSC 2011 8 (b) (i) 1

A bag contains seven balls numbered from 1 to 7. A ball is chosen at random and its number is noted. The ball is then returned to the bag. This is done a total of seven times.

What is the probability that each ball is selected exactly once?

Question 52: ME2 HSC 2011 8 (b) (ii) 1

A bag contains seven balls numbered from 1 to 7. A ball is chosen at random and its number is noted. The ball is then returned to the bag. This is done a total of seven times.

From part (i),

What is the probability that at least one ball is not selected?

Question 53: ME2 HSC 2011 8 (b) (iii) **2**

A bag contains seven balls numbered from 1 to 7. A ball is chosen at random and its number is noted. The ball is then returned to the bag. This is done a total of seven times.

From part (i),

From part (ii),

What is the probability that exactly one of the balls is not selected?

Question 54: ME1 HSC 2021 11 (d) **1**

A committee containing 5 men and 3 women is to be formed from a group of 10 men and 8 women.

In how many different ways can the committee be formed?

Question 55: ME1 HSC 2012 11 (e) **1**

In how many ways can a committee of 3 men and 4 women be selected from a group of 8 men and 10 women?

Question 56: ME1 HSC 2022 12 (b) **2**

A sports association manages 13 junior teams. It decides to check the age of all players. Any team that has more than 3 players above the age limit will be penalised.

A total of 41 players are found to be above the age limit.

Will any team be penalised? Justify your answer.

Question 57: ME1 HSC 2020 12 (c) **2**

To complete a course, a student must choose and pass exactly three topics.

There are eight topics from which to choose.

Last year 400 students completed the course.

Explain, using the pigeonhole principle, why at least eight students passed exactly the same three topics.

Question 58: ME1 HSC 2020 14 (a) (ii) 2

From part (i), you showed that

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2,$$

where n is a positive integer.

A club has $2n$ members, with n women and n men.

A group consisting of an even number $(0,\ 2,\ 4,\ \ldots,\ 2n)$ of members is chosen, with the number of men equal to the number of women.

Show, giving reasons, that the number of ways to do this is $\binom{2n}{n}$.

Question 59: ME1 HSC 2020 14 (a) (iii) 2

From part (i), you showed that

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \ldots + \binom{n}{n}^2,$$

where n is a positive integer.

A club has $2n$ members, with n women and n men.

A group consisting of an even number $(0,\ 2,\ 4,\ \ldots,\ 2n)$ of members is chosen, with the number of men equal to the number of women.

From part (ii), you showed that the number of ways to do this is $\binom{2n}{n}$.

From the group chosen in part (ii), one of the men and one of the women are selected as leaders.

Show, giving reasons, that the number of ways to choose the even number of people and then the leaders is

$$1^2\binom{n}{1}^2 + 2^2\binom{n}{2}^2 + \ldots + n^2\binom{n}{n}^2.$$

Question 60: ME1 HSC 2020 14 (a) (iv) 2

From part (i), you showed that

$$\binom{2n}{n}=\binom{n}{0}^2+\binom{n}{1}^2+\ldots+\binom{n}{n}^2,$$

where n is a positive integer.

A club has $2n$ members, with n women and n men.

A group consisting of an even number $(0,\ 2,\ 4,\ \ldots,\ 2n)$ of members is chosen, with the number of men equal to the number of women.

From part (ii), you showed that the number of ways to do this is $\binom{2n}{n}$.

From the group chosen in part (ii), one of the men and one of the women are selected as leaders.

From part (iii), you showed that the number of ways to choose the even number of people and then the leaders is

$$1^2\binom{n}{1}^2+2^2\binom{n}{2}^2+\ldots+n^2\binom{n}{n}^2.$$

The process is now reversed so that the leaders, one man and one woman, are chosen first. The rest of the group is then selected, still made up of an equal number of women and men.

By considering this reversed process and using part (ii), find a simple expression for the sum in part (iii).

Question 61: ME1 HSC 2018 14 (b) (ii) 2

From part (i), you showed that $\binom{n}{r}\binom{n}{r}+\binom{n}{r+1}\binom{r+1}{r}+\binom{n}{r+2}\binom{r+2}{r}+\ldots+\binom{n}{n}\binom{n}{r}=\binom{n}{r}2^{n-r}$ by considering the expansions of $\left(1+(1+x)\right)^n$ and $(2+x)^n$.

There are 23 people who have applied to be selected for a committee of 4 people.

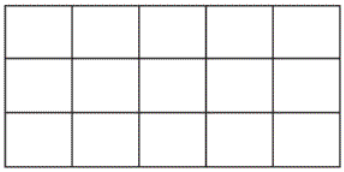
The selection process starts with Selector A choosing a group of at least 4 people from the 23 people who applied.

Selector B then chooses the 4 people to be on the committee from the group Selector A has chosen.

In how many ways could this selected process be carried out?

Question 62: ME2 HSC 2015 16 (a) (i) **2**

A table has 3 rows and 5 columns, creating 15 cells as shown.



Counters are to be placed randomly on the table so that there is one counter in each cell. There are 5 identical black counters and 10 identical white counters.

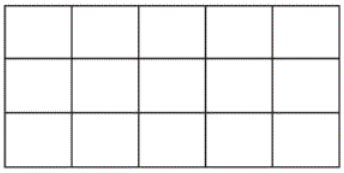
Show that the probability that there is exactly one black counter in each column is $\frac{81}{1001}$.

Question 63: ME2 HSC 2012 16 (a) (i) **1**

In how many ways can m identical yellow discs and n identical black discs be arranged in a row?

Question 64: ME2 HSC 2015 16 (a) (ii) **2**

A table has 3 rows and 5 columns, creating 15 cells as shown.



Counters are to be placed randomly on the table so that there is one counter in each cell. There are 5 identical black counters and 10 identical white counters.

From part (i), you showed that $P(\text{exactly one black in each column}) = \frac{81}{1001}$.

The table is extended to have n rows and q columns. There are nq counters, where q are identical black counters and the remainder are identical white counters. The counters are placed randomly on the table with one counter in each cell.

Let P_n be the probability that each column contains exactly one black counter.

Show that $P_n = \frac{n^q}{\binom{nq}{q}}$.

Question 65: ME2 HSC 2012 16 (a) (ii) **1**

In how many ways can 10 identical coins be allocated to 4 different boxes?