Q1

1a Factorisation

This is a standard factorisation you are expected to know,
$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$.
Hence,
$8x^3 + 27 = (2x)^3 + 3^3$
$= (2x + 3)((2x)^2 - 2x.3 + 3^2)$
$= (2x + 3)(4x^2 - 6x + 9)$
It does not factorise further (over the real numbers) since the discriminant $\Delta = (-6)^2 - 4.4.9 = -108 < 0$.

1b Domain

The function $y = \ln x$ has domain $x > 0$.
Hence the function $f(x) = \ln(x - 3)$ has domain $x - 3 > 0 \implies x > 3$

1c Trig Formula for small angles

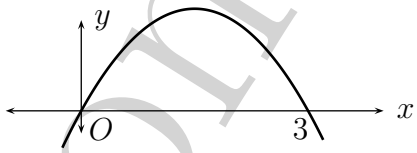
$$\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0} 2\frac{\sin 2x}{2x}$$
$$= 2\lim_{x\to 0} \frac{\sin 2x}{2x} = 2 \times 1 = 2$$

1d Solving Inequality.

These are usually best done by bringing everything to one side first, so that there is a zero on the right.

$$\begin{aligned} \frac{x+3}{2x} &> 1 \\ \frac{x+3}{2x} - 1 &> 0 \\ \frac{x+3-2x}{2x} &> 0 \\ \frac{3-x}{2x} &> 0 \\ (3-x)2x &> 0 \end{aligned}$$

By multiplying both sides by $(2x)^2$.
(We multiply both sides by a square here since by doing so the inequality symbol will not change, otherwise we have to consider different cases.)
Finally from a sketch of $y = 2x(3 - x)$ we see that y is positive when
$0 < x < 3$.



1e Differentiation

$$\begin{aligned} \tfrac{d}{dx}(x\cos^2 x) &= \cos^2 x + 2.x.\cos x \times (-\sin x) \\ &= \cos^2 x - 2x\cos x \sin x \\ &= \cos^2 x - x\sin 2x \end{aligned}$$

(using double angle formula)

1f Integration by substitution.

As suggested we use $u = x^3 + 1$, so that $du = 3x^2\,dx$
and noting that when $x = 2$, $u = 2^3+1 = 9$, and when $x = 0$, $u = 1$, so

$$\begin{aligned} I &= \int_0^2 x^2 e^{x^3+1}\,dx = \frac{1}{3}\int_1^9 e^u\,du \\ &= \frac{e^u}{3}\Big|_1^9 = \frac{e^9 - e}{3} \end{aligned}$$

Q2

2a Polynomials.

$p(x) = x^3 - ax + b$.

Recall the theory; that if a polynomial $p(x)$ has remainder r when divided by $x - a$ then $p(a) = r$.

So, $p(1) = 2$ and $p(-2) = 5$ from the information given in the question.

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Substituting into the polynomial we have,
$1^3 - a \times 12 + b = 2$ and $(-2)^3 - a(-2) + b = 5$ gives,
$b + 2a = 13 \quad (1)$
$b - a = 1 \quad (2)$

Subtracting these we get,
$3a = 12 \implies a = 4$
$\implies b - 4 = 1 \implies b = 5$
Hence $a = 4, b = 5$.

2b 2.b.i Auxiliary angle method
We need to solve for A and α where

$$3\sin x + 4\cos x = A\sin(x + \alpha)$$

$RHS = A\cos\alpha\sin x + A\sin\alpha\cos x$ and equating like terms we have
$A\cos\alpha = 3$ and $A\sin\alpha = 4$
$\therefore A = \sqrt{3^2 + 4^2} = 5$ (by squaring, adding, and using $\sin^2\alpha + \cos^2\alpha = 1$)
Also, $\tan\alpha = 4/3 \implies \alpha = \tan^{-1}(4/3)$ which is between 0 and $\pi/2$ since the argument $4/3$ is positive.

2.b.ii
$3\sin x + 4\cos x = 5$
$\therefore 5\sin\left(x + \tan^{-1}\frac{4}{3}\right) = 5$ (using part(i))
$\sin\left(x + \tan^{-1}\frac{4}{3}\right) = 1$
Hence, $x + \tan^{-1}\frac{4}{3} = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$
$\boxed{x = \frac{\pi}{2} - \tan^{-1}\frac{4}{3}}$ is the only solution in the correct range since $\frac{5\pi}{2} - \tan^{-1}\frac{4}{3} > 2\pi$.

2c Parametric form of parabola

2.c.i
$y = x^2/4$
$y' = 2x/4 = x/2$
So when $x = 2t$ then $y' = 2t/2 = t$, and hence the tangent at P is:
$y - t^2 = t(x - 2t)$
$y - t^2 = tx - 2t^2$
$y = tx - t^2$ as required.

2.c.ii
Tangents at Q: $y = 2tx - 4t^2$
(by putting $2t$ for t in the tangent at P)
So we solve the two equations;
$y = 2tx - 4t^2 \quad (1)$
$y = tx - t^2 \quad (2)$
$\therefore 2tx - 4t^2 = tx - t^2$
$tx = 3t^2$
$x = 3t$ (as $t \neq 0$ is assumed here)
$\implies y = t.3t - t^2 = 3t^2 - t^2 = 2t^2$
$\therefore R(3t, 2t^2)$

2.c.iii
We have $x = 3t, y = 2t^2$ and so we eliminate t by substituting $t = x/3$ into y;
$y = 2x^2/3^2$ and hence $y = \dfrac{2x^2}{9}$

Q3

3a $f(x) = \dfrac{3 + e^{2x}}{4}$

3.a.i Range

Start $e^x > 0$. (should be clear as its an exponential function)
$e^{2x} > 0$ (by squaring both sides, which you may do as both sides are non-negative)
$e^{2x} + 3 > 3$
$\dfrac{e^{2x} + 3}{4} > \dfrac{3}{4}$
Hence the range is $y > \dfrac{3}{4}$.

3.a.ii Inverse function
Firstly note that the inverse function does indeed exist as $f(x)$ is an increasing func-

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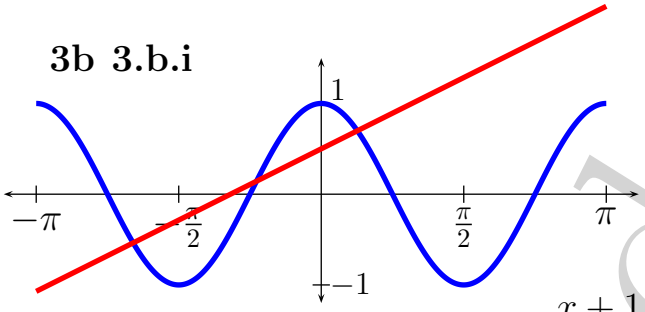
tion for all x, since $f'(x) = e^{2x}/2 > 0$.

The usual method to find the inverse function is to swap x and y in the expression for the function, then make y the subject and the resulting function is the inverse function.

$$x = \frac{3 + e^{2y}}{4}$$
$$3 + e^{2y} = 4x$$
$$e^{2y} = 4x - 3$$
$$\ln(e^{2y}) = \ln(4x - 3)$$
$$2y = \ln(4x - 3)$$
(since $\ln(e^x) = x$ is true for all x)

Therefore the inverse function is
$$y = \frac{1}{2}\ln(4x - 3)$$

3b 3.b.i



Graphs of $y = \cos 2x$ and $y = \dfrac{x+1}{2}$.

3.b.ii

$2\cos 2x = x + 1 \implies \cos 2x = \frac{x+1}{2}$ and hence from the graphs this has three solutions for $-\pi \le x \le \pi$.

3.b.iii Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \text{ (Newton's formula)}$$

$$f(x) = 2\cos 2x - x - 1$$
$$f'(x) = -4\sin 2x - 1$$

$$\therefore x_1 = 0.4 - \left(\frac{2\cos 0.8 - 0.4 - 1}{-4\sin 0.8 - 1}\right)$$
$x_1 = 0.398$ (3 d.p.)

3c 3.c.i

RTP: $\tan^2\theta = \dfrac{1 - \cos 2\theta}{1 + \cos 2\theta}$
(where $\cos 2\theta \neq -1$)

Recall that
$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ (basic trig identities)

Hence RHS $= \dfrac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta =$ LHS as required.

3.c.ii

$$\tan^2 \pi/8 = \frac{1 - \cos 2\pi/8}{1 + \cos 2\pi/8} = \frac{1 - \cos \pi/4}{1 + \cos \pi/4}$$
$$= \frac{1 - 1/\sqrt{2}}{1 + 1/\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$$
$$\therefore \tan \pi/8 = \sqrt{\frac{\sqrt{2} - 1}{\sqrt{2} + 1}}$$

where the positive square root was chosen since we know that $\tan \pi/8 > 0$
(Just consider the graph of $\tan x$ between 0 and $\pi/2$).

Q4

4a Binomial Probability

$p = 0.25$ (probability of success)
$q = 1 - p = 0.75$ (prob of failure)
$n = 5$ (the number of trials)
$r =$ (the number of successes)

4.a.i Here $r = 3$ and the required probability is
$$\binom{n}{r} p^r q^{n-r} = \binom{5}{3}(1/4)^3(3/4)^2 = \frac{45}{512}$$

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4.a.ii

$\binom{5}{3}(1/4)^3(3/4)^2 + \binom{5}{4}(1/4)^4(3/4)^1 + \binom{5}{5}(1/4)^5(3/4)^0$

$= \dfrac{\binom{5}{3}.9 + \binom{5}{4}.3 + \binom{5}{5}.1}{4^5}$

$= \dfrac{106}{1024} = \dfrac{53}{512}.$

4.a.iii At least one incorrect is the complement of none incorrect (ie all correct), so

$$\text{Prob} = 1 - \binom{5}{5}(1/4)^5(3/4)^0 = 1 - 1/1024 = 1023/1024$$

4b Curve Sketching

$$f(x) = \frac{x^4 + 3x^2}{x^4 + 3}$$

4.b.i

$$f(-x) = \frac{(-x)^4 + 3(-x)^2}{(-x)^4 + 3}$$

$= \dfrac{x^4 + 3x^2}{x^4 + 3} = f(x)$ (since $(-x)^4 = x^4$ and $(-x)^2 = x^2$; even negatives multiply to give positive)

Thus $f(-x) = f(x)$ and so the function is even.

4.b.ii $\lim\limits_{x\to\infty} \dfrac{x^4 + 3x^2}{x^4 + 3}$

$= \lim\limits_{x\to\infty} \dfrac{1 + 3/x^2}{1 + 3/x^4}$

(by dividing top/bottom by x^4, the highest power of x)

$= \dfrac{1 + 0^{++}}{1 + 0^{+}} = 1^{+}$

Therefore the horizontal asymptote is $y = 1$ (and this is also obvious as the coefficient of the highest power of x is 1.)

4.b.iii

Best to divide it out first

$$f(x) = \frac{x^4 + 3 + 3x^2 - 3}{x^4 + 3} = 1 + \frac{3x^2 - 3}{x^4 + 3}$$

$$f'(x) = \frac{(x^4 + 3).6x - (3x^2 - 3).4x^3}{(x^4 + 3)^2}$$

$$= \frac{6x^5 + 18x - 12x^5 + 12x^3}{(x^4 + 3)^2}$$

$$= -\frac{6x(x^4 - 2x^2 - 3)}{(x^4 + 3)^2}$$

Stationary points occur when $f'(x) = 0$ so solving we have

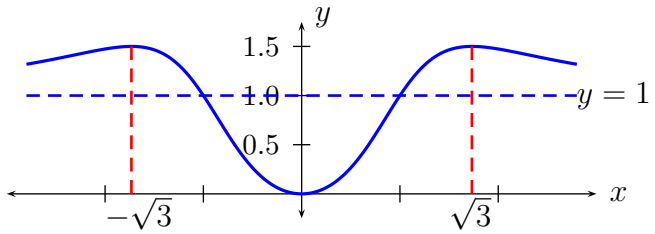
$x = 0$ or $x^4 - 2x^2 - 3 = 0 \implies (x^2 - 3)(x^2 + 1) = 0 \implies x = \pm\sqrt{3}$

Hence the stationary points are

$$(0, 0), (-\sqrt{3}, 3/2), (\sqrt{3}, 3/2)$$

(where we got the y-values by substituting into the function)

4.b.iv



Sketch of $y = f(x)$ where $f(x) = \dfrac{x^4 + 3x^2}{x^4 + 3}$.

Note that the function does cross the asymptote twice. (we can find them explicitly by solving $x^4 + 3x^2 = x^4 + 3 \implies x^2 = 1 \implies x \pm 1$.)

Q5

5a Simple Harmonic Motion

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5.a.i Recall that $\ddot{x} = \dfrac{d}{dx}\left(v^2/2\right)$ (assumed knowledge here)

Hence using this formula, and $\ddot{x} = -n^2x$, we have , $\dfrac{d}{dx}\left(v^2/2\right) = -n^2x$.
Integrating this gives us,
$v^2/2 = -n^2x^2/2 + c$ and since $x = a$ when $v = 0$ this allows us to find c;
$0 = -n^2a^2/2 + c \implies c = n^2a^2/2$.
Therefore $v^2/2 = -n^2x^2/2 + n^2a^2/2 \implies v^2 = -n^2x^2 + n^2a^2 = n^2(a^2 - x^2)$ as required.

(Note: The particle is in SHM about $x = 0$ and $-a \leq x \leq a$)

5.a.ii Maximum speed from (i) occurs when $a^2 - x^2$ is a maximum, and this occurs when x^2 is a minimum, that is, when $x = 0$, and so the maximum speed, $v^2 = n^2a^2 \implies v = na$.

(Note the speed refers to the absolute value of velocity.)

5.a.iii $\ddot{x} = -n^2x$ is greatest when $x = -a$ and so the maximum acceleration is n^2a.

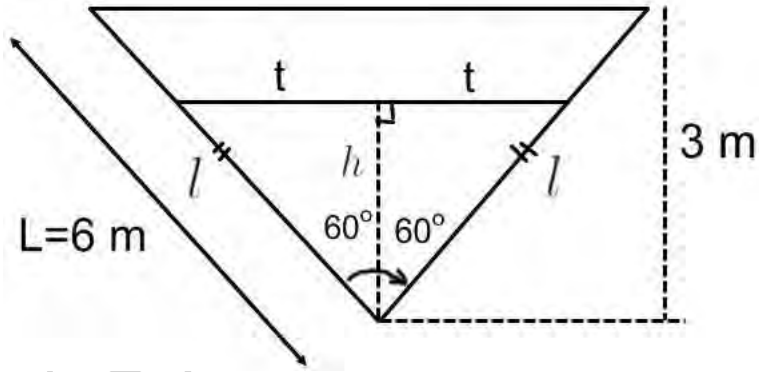
5.a.iv Since we are given that the particle is initially at the origin, then when $t = 0$, $x = 0$.
If we take the solution to be
$x = a\sin(nt + \alpha)$ then using the initial condition we obtain
$0 = a\sin\alpha \implies \alpha = 0$ and so updating,
$x(t) = a\sin nt$.

Velocity is $\dfrac{dx}{dt} = an\cos nt$

Therefore maximum speed is an (since max of $|\cos nt|$ is 1)
Half the maximum speed is $na/2$.
$an\cos nt = na/2 \implies 1/2 \implies nt = \pi/3 \implies t = \pi/3n$.

5b Maxima/Minima question.

The cross section is



5.b.i
$\cos 60^o = 3/L \implies L = 3/(1/2) = 6$m

By similar triangles (as we two corresponding angles equal) we have
$l/6 = h/3 \implies l = 2h$
$V = KH$ for a prism (where K is the cross sectional area and H is the length of the prism)

$K = 0.5ab\sin C$ (Area formula)
$K = 0.5 \times l \times l \times \sin 120^o = 0.5 \times 2h \times 2h \times \sqrt{3}/2$
$K = \sqrt{3}h^2$
$\therefore V = KH = \sqrt{3}h^2 \times 10 = 10\sqrt{3}h^2$

5.b.ii To find the top area we need to find the width t in the diagram above, using trig;
$\sin 60^o = t/l \implies t = l\sin 60^o = \sqrt{3}l/2 = \sqrt{3}h$
Area top surface

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$A = 10 \times 2t = 10 \times 2 \times \sqrt{3}h = 20\sqrt{3}h$ as required.

5.b.iii We are given
$\frac{dV}{dt} = -kA = -k \times 20\sqrt{3}h$
We regard V as a function of h and h as a function of t and use the chain rule;since $V = 10\sqrt{3}h^2$ then
$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = 20\sqrt{3}h\frac{dh}{dt}$
Equating the two expressions for dV/dt we have
$20\sqrt{3}h\frac{dh}{dt} = -k \times 20\sqrt{3}h$
$\frac{dh}{dt} = -k.$

5.b.iv We are given that when
$t = 0$, $h = 3$ and
$t = 100$, $h = 2$.
Since $\frac{dh}{dt} = -k$ this says that h changes at a constant rate so if it takes 100 days to go from 3 to 2m, then it will take 100 days to go from 2m to 1m.
So the time taken is 100 days.

Q6

6a Projectiles

6.a.i Equating the expressions given for x_1 and x_2 we have,
$UT\cos\theta = R - VT\cos\theta$
$T(U+V)\cos\theta = R$
$T = \frac{R}{(U+V)\cos\theta}$ as required.

6.a.ii They collide if both $x_1 = x_2$ and $y_1 = y_2$.
So we calculate $y_1 - y_2$ at the time when $x_1 = x_2$ (from (i))
$y_1 - y_2 = UT\sin\theta - gt^2/2 - h + VT\sin\theta + gt^2/2$
$= T(U+V)\sin\theta - h$
$= (U+V)\sin\theta \times \frac{R}{(U+V)\cos\theta} - h$
$= R\tan\theta - h$
$= R.h/R - h$
$= 0$ and thus $y_1 = y_2$ so we conclude that the projectiles collide at time T.

6.a.iii They collide at
$x = U\cos\theta \times \frac{R}{(U+V)\cos\theta} = \lambda R$
$\therefore \lambda(U+V) = U$
$\lambda U + \lambda V = U$
$\lambda V = U - \lambda U = U(1-\lambda)$
$V = U\left(\frac{1-\lambda}{\lambda}\right)$
$= U\left(\frac{1}{\lambda} - 1\right)$ as required

6b 6.b.i Geometric Series.

The geometric series

$$(1+x)^r + (1+x)^{r+1} + \cdots + (1+x)^n \qquad (*)$$

has ratio $r = (1+x)$, first term $a = (1+x)^r$, and number of terms is $n-r+1$, and hence the sum is
$S = \frac{(1+x)^r\left((1+x)^{n-r+1} - 1\right)}{1+x-1}$
$= \frac{(1+x)^{n+1} - (1+x)^r}{x}$
So we have

$$(1+x)^r + (1+x)^{r+1} + \cdots + (1+x)^n = \frac{(1+x)^{n+1} - (1+x)^r}{x}$$

We will compare coefficients of x^r on both sides and equate them to get the result.

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Coefficient of x^r in $(1+x)^r$ is $\binom{r}{r}$
Coefficient of x^r in $(1+x)^{r+1}$ is $\binom{r+1}{r}$
$\vdots$
Coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$

Hence on LHS the coefficient of x^r is $\binom{r}{r}+\binom{r+1}{r}+\cdots+\binom{n}{r}$

On the RHS:
The coefficient of x^{r+1} in $(1+x)^{n+1}$ is $\binom{n+1}{r+1}$.
The coefficient of x^{r+1} in $(1+x)^r$ is 0 (since x^r is the highest power here).

So, the coefficient of x^r on the RHS is $\binom{n+1}{r+1}$.
Hence
$$\binom{r}{r}+\binom{r+1}{r}+\cdots+\binom{n}{r}=\binom{n+1}{r+1},$$
as required.//

6.b.ii

Q6.b.ii.1 Since on the line $y=x$ there are n points and so we can choose 2 points in $\binom{n}{2}$ ways.
(each interval is defined by the two end-points of the interval)

Q6.b.ii.2 Walking from left to right and looking at the diagonals there are 2 points in the first diagonal, 3 in the second diagonal, 4 in the third,..., n,...,2. (We do not count just one point on the end as it does not define an interval)

And keeping in mind we need to find how many ways two points can be chosen in each diagonal, and using the formula in part (i) we have the total number of intervals is
$S_n=\binom{2}{2}+\binom{3}{2}+\cdots+\binom{n-1}{2}+\binom{n}{2}+\binom{n-1}{2}+\cdots+\binom{3}{2}+\binom{2}{2}$

6.b.iii

By (i) we have

$$\begin{aligned}S_n &= 2\left(\binom{2}{2}+\binom{3}{2}+\cdots+\binom{n-1}{2}\right)+\binom{n}{2}\\ &= 2\binom{n}{3}+\binom{n}{2}\\ &= \frac{2.n(n-1)(n-2)}{6}+\frac{n(n-1)}{2}\\ &= \frac{2.n(n-1)(n-2)+3n(n-1)}{6}\\ &= \frac{n(n-1)}{6}(2n-4+3)\\ &= \frac{n(n-1)(2n-1)}{6}\end{aligned}$$

as required.

Q7

7a

7.a.i Differentiation by first principles.

We have
$$\begin{aligned}\frac{d}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{x+h-x}{h}\\ &= \lim_{h\to 0}\frac{h}{h}\\ &= \lim_{h\to 0}1\\ &= 1\end{aligned}$$

7.a.ii Mathematical Induction.

Firstly, the statement for when $n=1$ was proven in part (i).
Secondly, we prove that the statement is true for $n=k+1$ whenever it is true for $n=k$.
So assume that $\frac{d}{dx}(x^k)=kx^{k-1}$ ...(*)
and prove that
$\frac{d}{dx}(x^{k+1})=(k+1)x^k$ ...(**).
Start, LHS of (**)$=\frac{d}{dx}(x^{k+1})=\frac{d}{dx}(x.x^k)=$

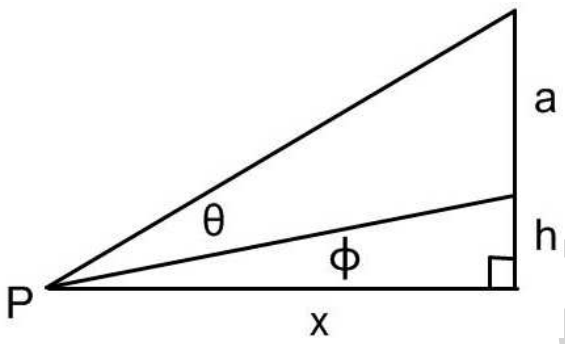
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$x\frac{d}{dx}(x^k)+x^k.\frac{d}{dx}(x)$ (using the product rule)
$= x.k.x^{k-1} + x^k.1$(invoking the induction hypothesis (*))
$= k.x^k + x^k = (k+1)x^k$=RHS of (**)
Hence we have shown that the statement is true when $n = k+1$ whenever it is true for $n = k$.
Finally, we conclude that by induction the statement is true for all positive integers n.

Remark: This is enough of a conclusion to a proof by induction—contrary to the advice of many textbooks. A reference is the 2007 examiners report, `http://www.boardofstudies.nsw.edu.au/hsc_exams/exam-papers-2007/pdf_doc/mathematics-ext2-notes-07.pdf`. The 2/3 unit syllabus confirms this view also - see page 47. JSH

7b



7.b.i

From the diagram using trig ratios,

$$\tan(\theta+\phi) = \frac{a+h}{x}$$
$$\tan\phi = h/x$$
$$\tan\theta = \tan(\theta+\phi-\phi)$$
$$= \frac{\tan(\theta+\phi)-\tan\phi}{1+\tan(\theta+\phi)\tan\phi}$$
$$= \frac{\frac{a+h}{x} - h/x}{1+\left(\frac{a+h}{x}\right)\times h/x}$$
$$= \frac{\frac{a+h-h}{x}}{\frac{x^2+h(a+h)}{x^2}}$$
$$= \frac{a}{x}.\frac{x^2}{x^2+h(a+h)}$$
$$\tan\theta = \frac{ax}{x^2+h(a+h)} \quad ...(\dagger)$$
$$\theta = \tan^{-1}\left(\frac{ax}{x^2+h(a+h)}\right) \quad ...(\ddagger)$$

7.b.ii

Differentiating (†) above with respect to x regarding θ as a function of x we have

$$\begin{aligned}\sec^2\theta\frac{d\theta}{dx} &= \frac{(x^2+h(a+h)).a - ax(2x)}{(x^2+h(a+h))^2} \\ &= \frac{ax^2+ah(a+h)-2ax^2}{(x^2+h(a+h))^2} \quad ...(\dagger\dagger) \\ &= \frac{ah(a+h)-ax^2}{(x^2+h(a+h))^2}\end{aligned}$$

(alternatively one could differentiate expression in (‡) directly to get the same answer)

$$\therefore \frac{d\theta}{dx} = 0 \implies x^2 = h(a+h)$$
$$\implies x = \sqrt{h(a+h)} \text{ (since } x > 0)$$

The sign change should be checked across the x-value but as the question asserts the existence of the maximum in this question we are done. (It's easy to check using , just factorise the numerator in (††) and it should be clear)

7c 7.c.i

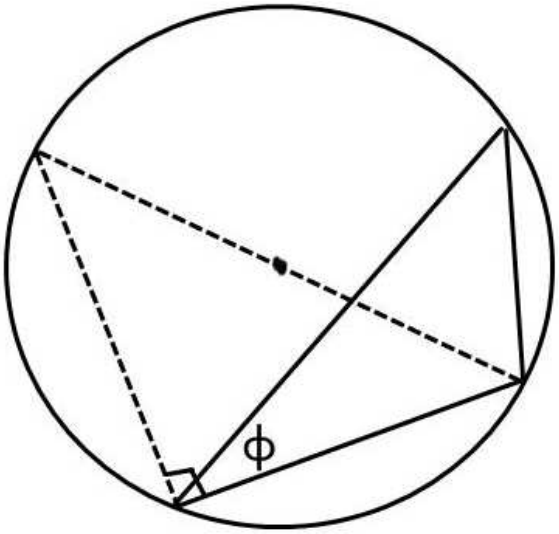
The circle must have centre to the left of the building in order that it has a tangent at T on the ground. (the centre is vertically above T).

The angle ϕ subtended by the minor segment with chord QR must be less than $\pi/2$ as can be seen from the diagram (the diam-

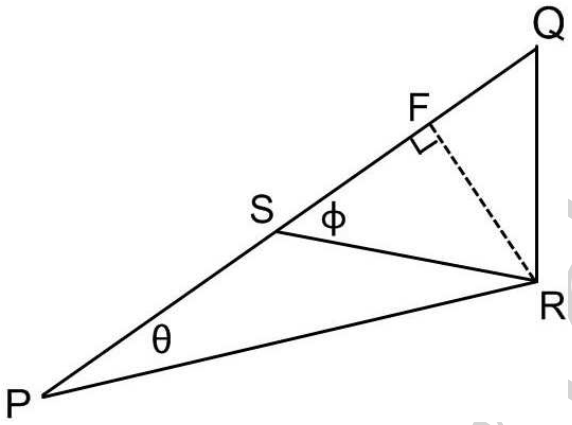
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eter subtends an angle of $\pi/2$ on the circle, and ϕ is less than this angle)



That is $\phi < \pi/2$.
Then dropping the perpendicular to F as shown below.



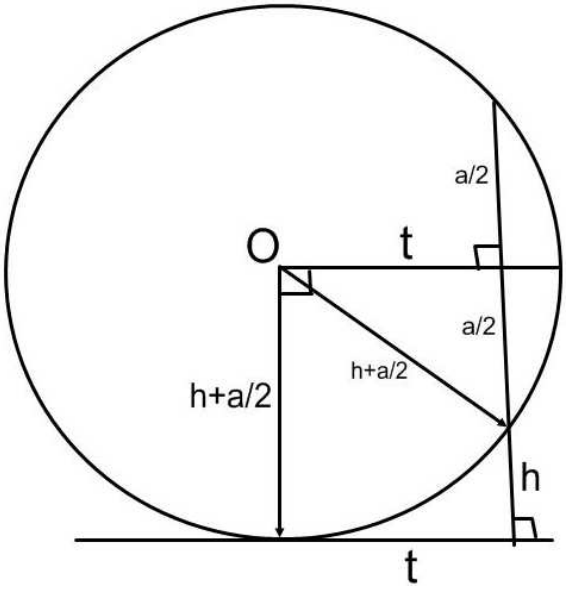
$\tan\phi = FR/SF$
$\tan\theta = FR/PF$
$\therefore \tan\theta/\tan\phi = \dfrac{FR/PF}{FR/SF}$
$= \dfrac{SF}{PF} = \dfrac{SF}{SF+PS} < 1$

If P and S are not the same point (and so P and T are different too) then $SF < PF$ so
$\tan\theta/\tan\phi < 1 \implies \tan\theta < \tan\phi \implies \theta < \phi$ (since $0 < \theta, \phi < \pi/2$ so the tangent function is increasing in that domain.)

If P and S are the same point (and so P and T are the same), then $SF = PF \implies \theta = \phi$.

As ϕ remains constant (angle on the circumference subtended by the chord QR) the θ attains the maximum value of ϕ when P, T are the same point.

7.c.ii



The point O is the centre of the circle. Using Pythagoras' theorem we have
$t^2 = (h + a/2)^2 - a^2/2^2$
$t^2 = h^2 + ha + a^2/4 - a^2/4$
$t = \sqrt{h^2 + ha}$

THE END.

Contact: Comments/corrections are appreciated, janh@hansendata.com.au,
Regards,
Jan Hansen 5 November 2009.

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