2001 HSC Full Solutions

Question 1 ☺ Question 2 ☺ Question 3 ☺
Question 4 ☺ Question 5 ☺ Question 6 ☺
Question 7 ☺



QUESTION 1

(a)

$$\int_0^2 \frac{dx}{\sqrt{16-x^2}} = \left[\sin^{-1}\frac{x}{4}\right]_0^2$$

$$= \sin^{-1}\frac{2}{4} - \sin^{-1}\frac{0}{4}$$

$$= \frac{\pi}{6}$$

(b)

$$\frac{d}{dx}x\sin^2 x = x.2\sin x.\cos x + \sin^2 x.1$$

$$= x\sin 2x + \sin^2 x$$

(c)

$$\sum_{n=4}^{7}(2n+3)$$

$$= (8+3)+(10+3)+(12+3)+(14+3)$$

$$= 56$$

(d)

$A(-2,7)$ $B(1,5)$

$1:-2$

$$x = \frac{1.1+-2.-2}{1-2} \quad y = \frac{1.5+-2.7}{1-2}$$

$$x = -5 \quad y = 9$$

$P(-5,9)$

(e)

$x+3$ will be a factor if $P(-3)=0$

$$P(-3) = (-3)^3 - 5(-3) + 12$$

$$= -27 + 15 + 12$$

$$= 0$$

Yes $x+3$ *is* a factor (Factor Theorem)

(f)

Let $u = 1+x$ *and* $x = u-1$

at x = -1, $u = 0$ & at $x = 0$, $u = 1$

$$\frac{du}{dx} = 1 \ \therefore du = dx$$

Now $15\int_{-1}^{0} x\sqrt{1+x}dx$

$$= 15\int_0^1 (u-1)\sqrt{u}.du$$

$$= 15\int_0^1 u^{\frac{3}{2}} - u^{\frac{1}{2}}.du$$

$$= 15\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{1}{2}u^{\frac{3}{2}}\right]_0^1 = 15\left(\frac{2}{5}-\frac{1}{2}\right) - 0 = -4$$

Question 2

(a)

$f(x) = 3x^2 + x$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{3(a+h)^2 + (a+h) - (3a^2 + a)}{h}$$

$$= \lim_{h\to 0} \frac{3(a^2 + 2ah + h^2) + a + h - 3a^2 - a}{h}$$

$$= \lim_{h\to 0} \frac{3a^2 + 6ah + 3h^2 + h - 3a^2}{h}$$

$$= \lim_{h\to 0} \frac{6ah + 3h^2 + h}{h}$$

$$= \lim_{h\to 0} \frac{h(6a + 3h + 1)}{h}$$

$$= \lim_{h\to 0} 6a + 3h + 1$$

$$= 6a + 1$$

(b) (i)

$$\int \frac{e^x}{1+e^x} dx = \ln(1+e^x) + C$$

This is in the form $\int \frac{f'(x)}{f(x)} dx$

See table of standard integrals

(b) (ii)

$$= \int_0^{\pi} \cos^2 3x dx$$

$$= \int_0^{\pi} \frac{1}{2}(1 + \cos 6x) dx$$

$$= \frac{1}{2}\left[x + \frac{1}{6}\sin 6x \right]_0^{\pi}$$

$$= \frac{1}{2}\left(\pi + \frac{1}{6}\sin 6\pi - 0 \right)$$

$$= \frac{1}{2}\pi$$

(c) (i)

9 letters with 2 the same

$$\frac{9!}{2!} = 181440$$

(c) (ii)

The 4 vowels, 2 alike can occupy

$\frac{4!}{2!} = 12$ ways

For the other 5 letters they can be arranged:

5!=120 ways

$\therefore$ Total arrangements

$12 \times 120 = 1440$

(d)	General term: ${}^{n}C_r(a)^{n-r}(b)^r$ ${}^{9}C_r(x^2)^{9-r}(-\frac{1}{x})^r$ ${}^{9}C_r x^{18-2r}(-1)^r x^{-r}$ ${}^{9}C_r x^{18-3r}(-1)^r$ The term independent of x i.e. x^0 will occur when 18-3$r = 0$ *i.e.* $r = 6$ So the term independent of x will be ${}^{9}C_6(-1)^6 = 84$

Question 3

(a) (i)

$f(x) = \sin x + \cos x - x$

$f'(x) = \cos x - \sin x - 1$

$x_2 = x_1 - \dfrac{f(x)}{f'(x)}$

$x_2 = 1.2 - \dfrac{f(1.2)}{f'(1.2)}$

$= 1.2 - \dfrac{0.0943}{-1.5697}$

$= 1.26$ to 3 sig. fig.

(b) (i)

See diagram on next page

$\angle AOB = 2\theta$ (angle at centre is twice the angle on the circumference subtending the same arc)

(b) (ii)

$\angle TAB = \angle AOB$ (alternate angle theorem, circle $C1$)

(b) (iii)

$\angle PBA = 2\theta - \theta = \theta$ (exterior angle = sum of 2 interior angles)

$\Delta \therefore APB$ is isosceles.

$\therefore PA = BA$

(c) (i)

$= \sin 3\theta$

$= \sin(\theta + 2\theta)$

$= \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$

$= \sin\theta(1 - 2\sin^2\theta) + \cos\theta \times 2\sin\theta\cos\theta$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta\cos^2\theta$

$= \sin\theta - 2\sin^3\theta + 2\sin\theta(1 - \sin^2\theta)$

$= 3\sin\theta - 4\sin^3\theta.$

(c) (ii)

$\sin 3\theta = 2\sin\theta$ is equivalent to solving

$3\sin\theta - 4\sin^3\theta = 2\sin\theta$

$\sin\theta - 4\sin^3\theta = 0$

$\sin\theta(1 - 4\sin^2\theta) = 0$

$\therefore \sin\theta = 0, \sin^2\theta = \dfrac{1}{4}$

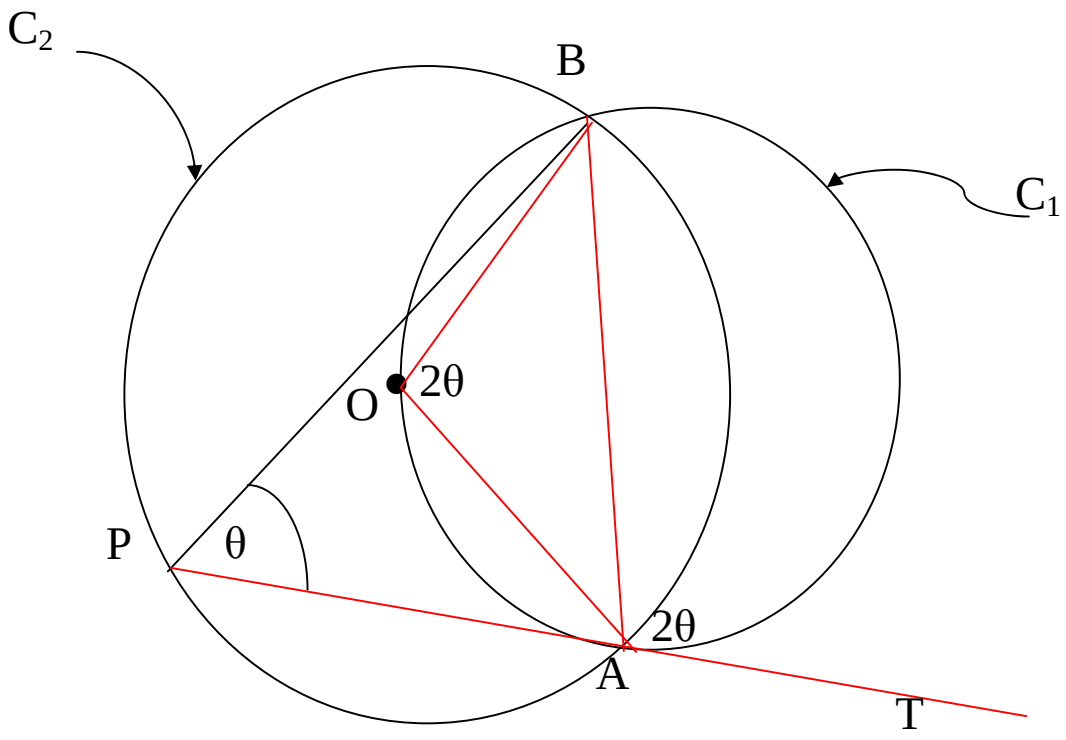
$\therefore \sin\theta = 0, \sin\theta = \pm\dfrac{1}{2}$

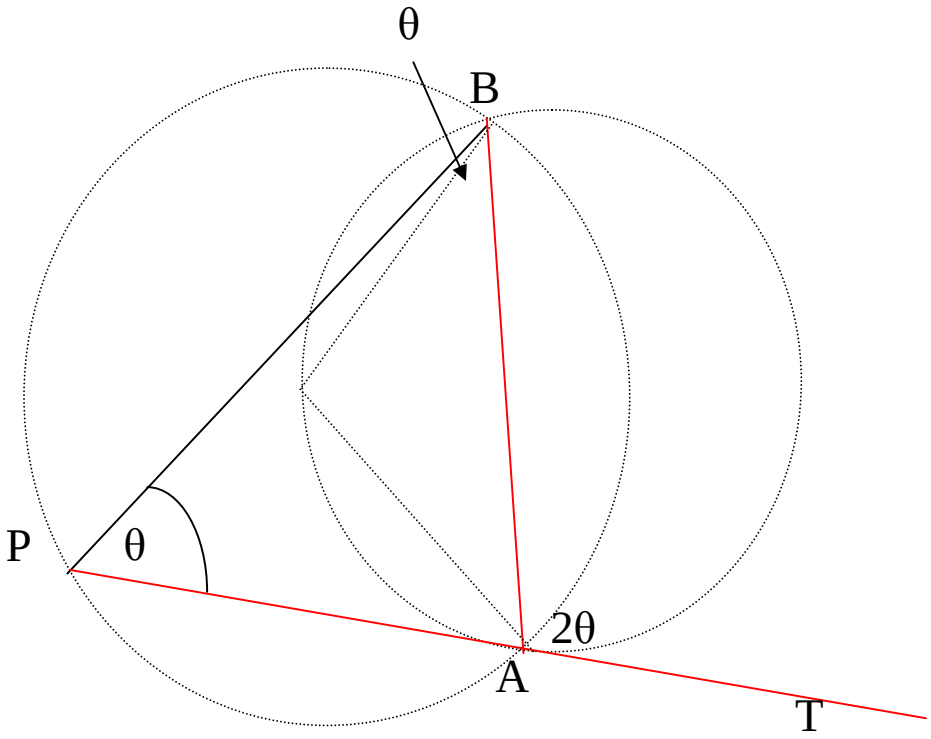
Solving $\sin\theta = 0$ gives $\theta = 0, \pi, 2\pi.$

Solving $\sin\theta = \dfrac{1}{2}$ gives $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Solving $\sin\theta = -\dfrac{1}{2}$ gives $\theta = -\dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}.$

$\therefore \theta = 0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \pi, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, 2\pi.$


C2
B
C1
O
2θ
P
θ
2θ
A
T


θ
B
P
θ
2θ
A
T

Question 4

(a) (i)

$$\frac{3x}{x-2} \le 1$$

$$(x-2)^2 \frac{3x}{x-2} \le 1.(x-2)^2$$

since $\times$ both sides by a +ve number don't need worry about inequality

$$3x(x-2) \le (x-2)^2$$

$$3x(x-2)-(x-2)^2 \le 0$$

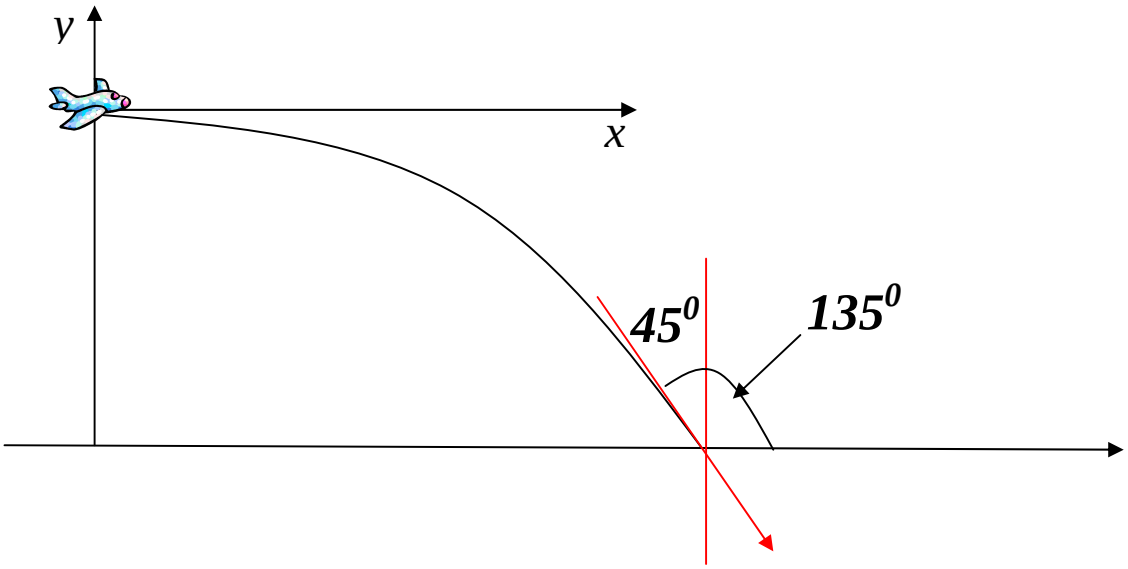
$$(x-2)(3x-(x-2)) \le 0$$

$$(x-2)(2x+2) \le 0$$

$$2(x-2)(x+1) \le 0$$

$$-1 \le x < -2 \quad note \quad x \ne 2$$

(b)



Given $x = Vt \quad y = -5t^2 \qquad \therefore \qquad \dot{x} = V \quad \dot{y} = -10t.$

At the point of impact $\tan(135^0) = \frac{\dot{y}}{\dot{x}}$,

$$-1 = \frac{-10t}{V}. \qquad \therefore \qquad t = \frac{V}{10}.$$

Also at the point of impact $x = 4000$ m.

$\therefore x = Vt$

$$4000 = V.\frac{V}{10} = \frac{V^2}{10}$$

$V^2 = 40000 \qquad V = 200$ m/s.

(c)(i)

$\ddot{x} = -4x$

$\therefore \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$

$\therefore \frac{1}{2}v^2 = \int -4x.dx$

$\frac{1}{2}v^2 = -2x^2 + C_1$

$v^2 = -4x^2 + C_2$

Given $x = 3 \;\; v = -6\sqrt{3}$

$\therefore \left(-6\sqrt{3}\right)^2 = -4.3^2 + C_2$

$C_2 = 144$

$\therefore v^2 = -4x^2 + 144$

$v = \pm\sqrt{144 - 4x^2}$

$v = \pm 2\sqrt{36 - x^2}$

However, since v= $-6\sqrt{3}$ when x=3

then v is -ve

$\therefore v = -2\sqrt{36 - x^2}$

$\therefore \frac{dx}{dt} = -2\sqrt{36 - x^2}$

$\therefore \frac{dt}{dx} = \frac{1}{-2\sqrt{36 - x^2}}$

$\therefore t = -\frac{1}{2}\int \frac{1}{\sqrt{36 - x^2}}.dx$

$t = -\frac{1}{2}\sin^{-1}\frac{x}{6} + C_3$

Given $t = 0$ *when* $x = 3$

$\therefore 0 = -\frac{1}{2}\sin^{-1}\frac{3}{6} + C_3$

$0 = -\frac{1}{2}\frac{\pi}{6} + C_3$

$C_3 = \frac{\pi}{12}$

$\therefore t = -\frac{1}{2}\sin^{-1}\frac{x}{6} + \frac{\pi}{12}$

$2t = -\sin^{-1}\frac{x}{6} + \frac{\pi}{6}$

$\sin^{-1}\frac{x}{6} = \frac{\pi}{6} - 2t$

$\frac{x}{6} = \sin\left(\frac{\pi}{6} - 2t\right)$

$x = 6\sin\left(\frac{\pi}{6} - 2t\right)$

Question 5

(a) (i)

(i) Let $x = 0$

$$y = 2\cos^{-1} 0$$

$$y = 2 \times \frac{\pi}{2}$$

$$y = \pi$$

(a) (ii)

(ii) $f : y = 2\cos^{-1}\frac{x}{3}, -3 \le x \le 3, 0 \le y \le 2\pi$

$$f^{-1} : x = 2\cos^{-1}\frac{y}{3}$$

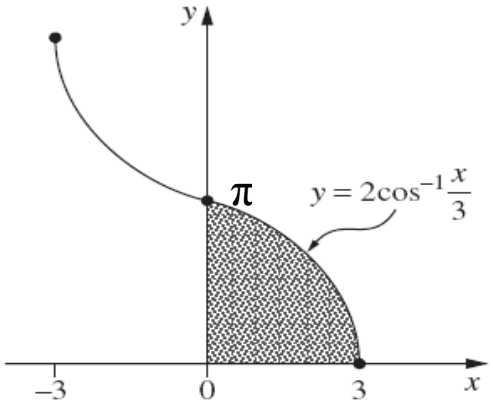
$$\frac{x}{2} = \cos^{-1}\frac{y}{3}$$

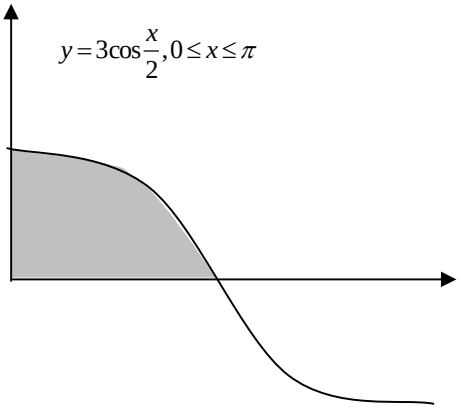
$$\frac{y}{3} = \cos\frac{x}{2}$$

$$y = 3\cos\frac{x}{2}, 0 \le x \le 2\pi$$

(a) (iii)

The shaded area is the same as the area under the curve





$$\therefore \int_0^{\pi} 3\cos\frac{x}{2}\,dx = \left[6\sin\frac{x}{2}\right]_0^{\pi} = 6 \text{ units}^2$$

(b)
(ii)

$$(q+p)^n = \binom{n}{0}q^n + \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 + \binom{n}{3}q^{n-3}p^3 + ...(1)$$

$$(q-p)^n = \binom{n}{0}q^n - \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 - \binom{n}{3}q^{n-3}p^3 + ...(2)$$

$(1)-(2)$

$$(q+p)^n - (q-p)^n = 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3 + ...(3)$$

In the subtraction above, notice that only the even terms stay,
also knowing that there are (n + 1) terms in each expansion,
we can conclude that:

- if n is odd, the last term in (3) is twice the last term of (1),

i.e. $2\binom{n}{n}p^n = 2p^n$

- if n is even, the last term in (3) is twice the second last term of (1),

i.e. $2\binom{n}{n-1}qp^{n-1} = 2nqp^{n-1}$

(c)(i)

Let $p = \Pr(\text{six}) = \frac{1}{6}, q = \Pr(\text{non 6}) = \frac{5}{6}.$

Pr(exactly *r* sixes) = $\binom{n}{r}\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{n-r}$

(c)(ii)

Pr(an odd number of sixes) = Pr(1 six) + Pr(3 sixes) + …

From part (a), $(q+p)^n - (q-p)^n = 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3 + ...$

$$\binom{n}{1}q^{n-1}p + \binom{n}{3}q^{n-3}p^3 + ... = \frac{(q+p)^n - (q-p)^n}{2}.$$

Take $p = \Pr(\text{six}) = \frac{1}{6}, q = \Pr(\text{non 6}) = \frac{5}{6},$

$$\binom{n}{1}\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right) + \binom{n}{3}\left(\frac{5}{6}\right)^{n-3}\left(\frac{1}{6}\right)^3 + ... = \frac{\left(\frac{5}{6}+\frac{1}{6}\right)^n - \left(\frac{5}{6}-\frac{1}{6}\right)^n}{2} = \frac{1}{2}\left(1-\left(\frac{2}{3}\right)^n\right)$$

Question 6

(a)

Step 1: Show true for $n=1$

$LHS = 1^3 + (1+1)^3 + (1+2)^3$

$= 1+8+27 = 36$

$36 \div 9 = 4$

True for n=1

Step 2: Assume true for say k

i.e. $k^3 + (k+1)^3 + (k+2)^3 = 9p$ where p is an integer

or this may be written as $(k+1)^3 + (k+2)^3 = 9p - k^3$

Show true for k+1

i.e. $(k+1)^3 + (k+2)^3 + (k+3)^3 = 9m$ where m is an integer

$LHS = (k+1)^3 + (k+2)^3 + (k+3)^3$

$= 9p - k^3 + (k+3)^3$

$= 9p - k^3 + k^3 + 9k^2 + 27k + 27$

$= 9p + 9k^2 + 27k + 27$

$= 9(p + k^2 + 3k + 3)$

$= 9m$

where $m = p + k^2 + 3k + 3$

Step 3: Since true for n=1 (Step 1) then true for n=2 (Step 2)

Since true for n=2 now true for n=3 and so on. Hence by

mathematical induction it is true for all positive integers n.

(b) (i)

Gradient of the tangent at P is t

$\therefore$ The gradient of the normal at P is $-\frac{1}{t}$

Equation of normal is $y - at^2 = -\frac{1}{t}(x - 2at)$

$ty - at^3 = -x + 2at$

$x + ty = at^3 + 2at$

(b) (ii)	Let Q have co-ordinates $(2aq, aq^2)$ $\therefore$ Gradient of the normal at Q = $-\frac{1}{q}$ Since this normal is perpendicular to PR with gradient $-\frac{1}{t}$ then $-\frac{1}{q}\times-\frac{1}{t}=-1$ $\frac{1}{qt}=-1$ $q=-\frac{1}{t}$ $\therefore Q=\left(\frac{-2a}{t},\frac{a}{t^2}\right)$

(b) (iii)

Gradient of RP = $-\frac{1}{t}$

$\therefore$ Gradient of QR = t

Equation of QR: $y-\frac{a}{t^2}=t\left(x-\left(\frac{-2a}{t}\right)\right)$

$$y-\frac{a}{t^2}=tx+2a$$

$$\therefore y=tx+2a+\frac{a}{t^2}$$

Solve simultaneously to find R

$y=tx+2a+\frac{a}{t^2}$.....(1)

$x+ty=2at+at^3$.....(2)

sub (1) into (2)

$x+t\left(tx+2a+\frac{a}{t^2}\right)=2at+at^3.$

$x+t^2x+2at+\frac{a}{t}=2at+at^3$

$x(1+t^2)+\frac{a}{t}=at^3$

$x(1+t^2)=at^3-\frac{a}{t}$

$x=\frac{at^3-\frac{a}{t}}{(1+t^2)}$

$=\frac{a\left(t^3-\frac{1}{t}\right)}{(1+t^2)}$

$=\frac{a\left(t^4-1\right)}{t(1+t^2)}$

$=\frac{a\left(t^2-1\right)\left(t^2+1\right)}{t(1+t^2)}\quad=\frac{a\left(t^2-1\right)}{t}$

$x=a\left(t-\frac{1}{t}\right)$

sub x into (1) $y=t\left(a\left(t-\frac{1}{t}\right)\right)+2a+\frac{a}{t^2}$

$y=at^2-a+2a+\frac{a}{t^2}=at^2+a+\frac{a}{t^2}$

$\therefore y=a\left(t^2+1+\frac{1}{t^2}\right)$

(b) (iv)	Need to write in Cartesian Form $x = a\left(t - \frac{1}{t}\right)$ $y = a\left(t^2 + 1 + \frac{1}{t^2}\right) = a\left(t^2 + \frac{1}{t^2} + 1\right)$ Use $\left(t - \frac{1}{t}\right)^2 = t^2 + \frac{1}{t^2} - 2,$ since $t - \frac{1}{t} = \frac{x}{a}$ and $t^2 + \frac{1}{t^2} = \frac{y}{a} - 1$ $\therefore \frac{x^2}{a^2} = \frac{y}{a} - 1 - 2$ $\therefore \frac{x^2}{a^2} = \frac{y}{a} - 3$ $\therefore x^2 = ay - 3a^2$

Question 7

(a) (i)

$$\frac{dv}{dt} = x - 1$$

$$a = x - 1$$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 1$$

$$\frac{1}{2}v^2 = \int x - 1.dx$$

$$\frac{1}{2}v^2 = \frac{1}{2}x^2 - x + C$$

When $x = 0\ v = 1$

$$\frac{1}{2} = C$$

$$\frac{1}{2}v^2 = \frac{1}{2}x^2 - x + \frac{1}{2}$$

$$v^2 = x^2 - 2x + 1$$

$$v^2 = (x-1)^2$$

(a) (ii)

$$v^2 = (x-1)^2 \text{ or } \left(\frac{dx}{dt}\right)^2 = (x-1)^2$$

$$\therefore \frac{dx}{dt} = \pm(x-1)$$

Since at $t = 0, x = 0, \frac{dx}{dt} = 1$

$$\therefore 1 = \pm(0-1) \quad \text{must be -}$$

$$\therefore \frac{dx}{dt} = -(x-1) = 1 - x$$

$$\therefore \frac{dt}{dx} = \frac{1}{1-x}$$

$$\therefore t = \int \frac{1}{1-x} dx$$

$$t = -\ln(1-x) + C_1$$

at $t = 0, x = 0$

$$\therefore 0 = -\ln(1-0) + C_1$$

$$\therefore C_1 = 0$$

$$\therefore t = -\ln(1-x)$$

$$\ln(1-x) = -t$$

$$(1-x) = e^{-t}$$

$$x = 1 - e^{-t}$$

(b) (i)

$AP^2 = AO^2 + OP^2 - 2(AO)(OP)\cos\frac{\pi}{3}$

From $\Delta AOT \quad AO = h$ *– isosceles triangle*

From $\Delta TOP \quad OP = \frac{h}{\tan\alpha} = h\cot\alpha$

$\therefore AP^2 = h^2 + (h\cot\alpha)^2 - 2(h)(h\cot\alpha)\cos\frac{\pi}{3}$

$= h^2 + h^2\cot^2\alpha - 2h^2\cot\alpha.\frac{1}{2}$

$= h^2 + h^2\cot^2\alpha - h^2\cot\alpha$

(b) (ii)

In ΔATP

$AP^2 = AT^2 + TP^2 - 2(AT)(TP)\cos\theta$

In ΔOAT

$\sin\frac{\pi}{4} = \frac{h}{AT}$

$\therefore AT = h\sqrt{2}$

In ΔPTO

$\sin\alpha = \frac{h}{TP}$

$\therefore TP = h\cos ec\alpha$

$AP^2 = (h\sqrt{2})^2 + (h\cos ec\alpha)^2 - 2(h\sqrt{2})(h\cos ec\alpha)\cos\theta$

$AP^2 = 2h^2 + h^2\cos ec^2\alpha - 2\sqrt{2}h^2\cos ec\alpha\cos\theta$

From (i) $AP^2 = h^2 + h^2\cot^2\alpha - h^2\cot\alpha$

$\therefore h^2 + h^2\cot^2\alpha - h^2\cot\alpha = 2h^2 + h^2\cos ec^2\alpha - 2\sqrt{2}h^2\cos ec\alpha\cos\theta$

$1 + \cot^2\alpha - \cot\alpha = 2 + \cos ec^2\alpha - 2\sqrt{2}\cos ec\alpha\cos\theta \quad .....(\div h^2)$

$-\cot\alpha = 2 - 2\sqrt{2}\cos ec\alpha\cos\theta$

$2\sqrt{2}\cos ec\alpha\cos\theta = 2 + \cot\alpha$

$\frac{2\sqrt{2}\cos\theta}{\sin\alpha} = 2 + \frac{\cos\alpha}{\sin\alpha}$

$\cos\theta = \frac{2\sin\alpha}{2\sqrt{2}} + \frac{\cos\alpha}{\sin\alpha}.\frac{\sin\alpha}{2\sqrt{2}}$

$\cos\theta = \frac{\sin\alpha}{\sqrt{2}} + \frac{\cos\alpha}{2\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$

(b) (iii)

From (ii) $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$

$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)$

$$\frac{d\theta}{d\alpha} = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)^2}} \times \left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{2\sqrt{2}}\sin\alpha\right)$$

$$= \frac{-1}{\sqrt{1-\left(\frac{2\sin\alpha+\cos\alpha}{2\sqrt{2}}\right)^2}} \times \left(\frac{2\cos\alpha-\sin\alpha}{2\sqrt{2}}\right)$$

$$= \frac{\sin\alpha - 2\cos\alpha}{\sqrt{\left(\frac{\left(2\sqrt{2}\right)^2 - (2\sin\alpha+\cos\alpha)^2}{\left(2\sqrt{2}\right)^2}\right)}} \times \left(\frac{1}{2\sqrt{2}}\right)$$

$$= \frac{\sin\alpha - 2\cos\alpha}{\frac{\sqrt{8-(2\sin\alpha+\cos\alpha)^2}}{2\sqrt{2}}} \times \frac{1}{2\sqrt{2}}$$

$$= \frac{\sin\alpha - 2\cos\alpha}{\sqrt{8-(2\sin\alpha+\cos\alpha)^2}}$$

$\frac{d\theta}{d\alpha} = 0$ gives possible turning points

$$\therefore 0 = \frac{\sin\alpha - 2\cos\alpha}{\sqrt{8-(2\sin\alpha+\cos\alpha)^2}}$$

$0 = \sin\alpha - 2\cos\alpha$

$-\sin\alpha = -2\cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = 2 = \tan\alpha \therefore \alpha \approx 1.1$

If $\tan\alpha = \frac{2}{1} \therefore$ *hypotenuse* $= \sqrt{5} \therefore \sin\alpha = \frac{2}{\sqrt{5}}, \cos\alpha = \frac{1}{\sqrt{5}}$

Now $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{2}}\frac{1}{\sqrt{5}}\right) \approx 0.66$

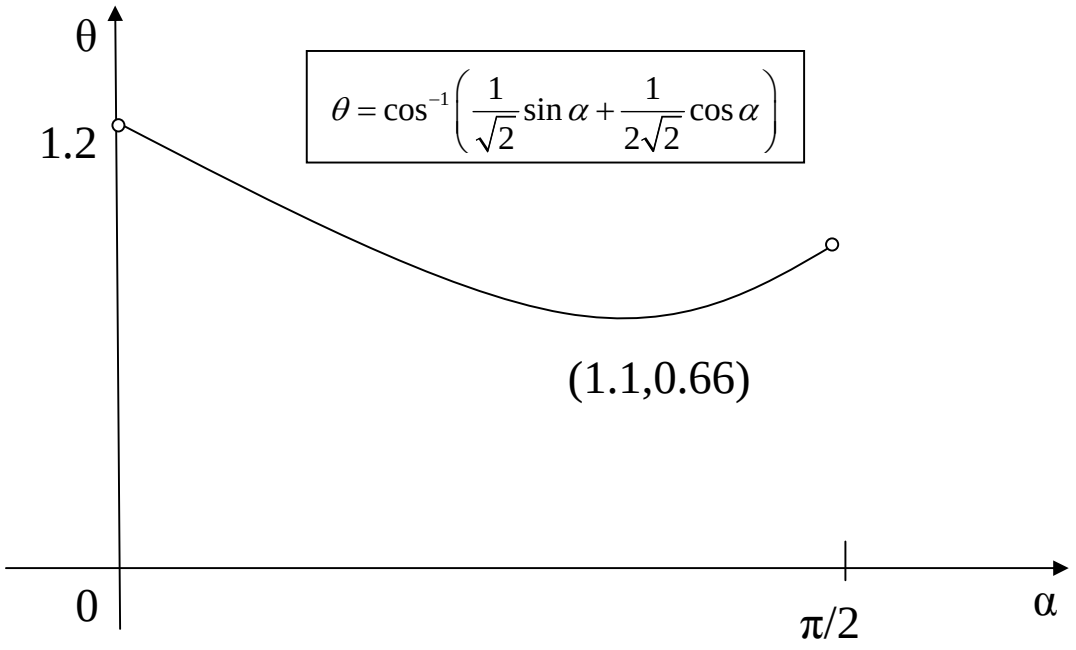
$\therefore$ turning point at $(1.1, 0.66)$

Consider sign change with $\frac{d\theta}{d\alpha}$ to find either max. or min. point.

As $\alpha \to 0, \frac{d\theta}{d\alpha} \to \frac{-2}{\sqrt{7}} < 0$

As $\alpha \to \frac{\pi}{2}, \frac{d\theta}{d\alpha} \to \frac{1}{2} > 0 \therefore$ a minimum turning point.

As $\alpha \to 0, \cos\theta \to \frac{1}{2\sqrt{2}} \therefore \theta \to 1.2$ *and* As $\alpha \to \frac{\pi}{2}, \cos\theta \to \frac{1}{\sqrt{2}} \therefore \theta \to \frac{\pi}{4}$


θ
1.2
$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)$
(1.1,0.66)
0
π/2
α