2001 Higher School Certificate Solutions

MATHEMATICS EXTENSION 1

QUESTION 1

(a) $\displaystyle\int_0^2 \frac{dx}{\sqrt{16-x^2}} = \left[\sin^{-1}\frac{x}{4}\right]_0^2$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1} 0$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}.$

(b) $\dfrac{d}{dx}\left(x\sin^2 x\right) = 1\cdot\sin^2 x + x\cdot 2\sin x\cos x$

$= \sin^2 x + 2x\sin x\cos x.$

(c) $\displaystyle\sum_{n=4}^{7}(2n+3) = (2\times 4+3)+(2\times 5+3)$

$+(2\times 6+3)+(2\times 7+3)$

$= 11+13+15+17$

$= 56.$

(d) $A(-2, 7)$ $B(1, 5)$

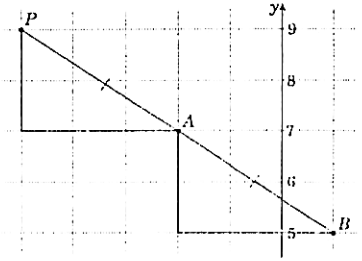
$-1 : 2$

P has coordinates

$\left(\dfrac{2\times(-2)+(-1)\times 1}{2-1}, \dfrac{2\times 7+(-1)\times 5}{2-1}\right)$

$= (-5, 9).$

Alternatively, an external ratio of $1 : 2$ means that $PA = AB$, and the result can be found easily from a diagram.



(e) Let $P(x) = x^3 - 5x + 12$

$P(-3) = (-3)^3 - 5(-3) + 12$

$= 0.$

$\therefore$ $x+3$ is a factor of $P(x)$ by the Factor Theorem.

OR

$$x+3\overline{)\,x^2 - 3x + 4}$$

$$x^3 \quad -5x+12$$

$$x^3+3x^2$$

$$-3x^2-5x$$

$$-3x^2-9x$$

$$4x+12$$

$$4x+12$$

$$0$$

$\therefore$ $x^3 - 5x + 12 = (x+3)\left(x^2 - 3x + 4\right)$

ie. $x+3$ is a factor.

(f) $u = 1 + x$

$du = dx.$

$x = -1 \quad u = 0$

$x = 0 \quad u = 1.$

$\therefore$ $\displaystyle 15\int_{-1}^{0} x\sqrt{1+x}\ dx$

$\displaystyle = 15\int_0^1 (u-1)\sqrt{u}\ du$

$\displaystyle = 15\int_0^1 \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$

$\displaystyle = 15\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$

$\displaystyle = 15\left[\left(\frac{2}{5} - \frac{2}{3}\right) - 0\right]$

$= -4.$

Note that the integral is negative since $x\sqrt{1+x} \le 0$ on the domain $-1 \le x \le 0$.

QUESTION 2

(a) $f(x) = 3x^2 + x$

$f'(a) = \displaystyle\lim_{h\to 0}\frac{f(a+h) - f(a)}{h}$

$\displaystyle = \lim_{h\to 0}\frac{\left[3(a+h)^2 + (a+h)\right] - \left(3a^2 + a\right)}{h}$

$\displaystyle = \lim_{h\to 0}\frac{3a^2 + 6ah + 3h^2 + a + h - 3a^2 - a}{h}$

$\displaystyle = \lim_{h\to 0}\frac{6ah + 3h^2 + h}{h}$

$\displaystyle = \lim_{h\to 0}\left(6a + 3h + 1\right)$

$= 6a + 1.$

(b) (i) $\displaystyle\int \frac{e^x}{1+e^x}\ dx = \ln\left(1+e^x\right) + c.$

(ii) $\displaystyle\int_0^{\pi}\cos^2 3x\ dx$

$\displaystyle = \frac{1}{2}\int_0^{\pi}(1+\cos 6x)\ dx$

$\displaystyle = \frac{1}{2}\left[x + \frac{\sin 6x}{6}\right]_0^{\pi}$

$\displaystyle = \frac{1}{2}\left[\left(\pi + \frac{\sin 6\pi}{6}\right) - \left(0 + \frac{\sin 0}{6}\right)\right]$

$= \frac{\pi}{2}.$

Note $\displaystyle\frac{1}{2}\int_0^{\pi}(1+\cos 6x)\ dx = \frac{1}{2}\int_0^{\pi} 1\ dx,$

since $\cos 6x$ has a period of $\frac{\pi}{3}$ and the integration is over 3 complete periods.

(c) There are 2As.

The other 7 letters are unique.

(i) The number of arrangements $= \dfrac{9!}{2!}$

$= 181\,440.$

(ii) The number of arrangements of vowels $= \dfrac{4!}{2!}.$

The number of arrangements of consonants $= 5!.$

Total number of arrangements $= \dfrac{4!}{2!}\times 5!$

$= 1440.$

(d) In the expansion of $\left(x^2 - \dfrac{1}{x}\right)^9$,

$T_{k+1} = \binom{9}{k}\left(x^2\right)^{9-k}\left(-\frac{1}{x}\right)^k$

$= \binom{9}{k}(-1)^k x^{18-3k}.$

For term independent of x, $18 - 3k = 0$

$k = 6.$

$\therefore$ Required term $= \binom{9}{6}(-1)^6$

$= \binom{9}{6}$

$= 84.$

QUESTION 3

(a) $f(x) = \sin x + \cos x - x$

$f'(x) = \cos x - \sin x - 1$

$f(1{\cdot}2) = 0{\cdot}094\ 39\ \ldots$

$f'(1{\cdot}2) = -1{\cdot}569\ 68\ \ldots$

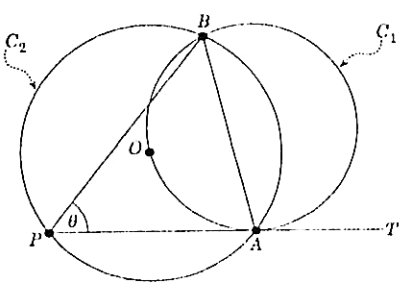
Let x_2 be a second approximation:

$x_2 = 1{\cdot}2 - \dfrac{f(1{\cdot}2)}{f'(1{\cdot}2)}$ by Newton's method

$= 1{\cdot}2 - \dfrac{0{\cdot}094\ 39\ \ldots}{-1{\cdot}569\ 68\ \ldots}$

$= 1{\cdot}26$ (3 sig. figs).

(b)



(i) $\angle AOB = 2\theta$ (Angle at the centre is twice angle at the circumference standing on the same arc.)

(ii) $\angle TAB$ is the angle between the tangent to C_1 and the chord AB.

$\angle AOB$ is the $\angle$ in the alternate segment.

$\therefore$ $\angle TAB = \angle AOB = 2\theta.$

(iii) $\angle TAB = \angle APB + \angle ABP$ (Ext. $\angle$ of a Δ = sum of int. opp. $\angle$s)

$2\theta = \theta + \angle ABP.$

$\therefore \angle ABP = \theta$

$\therefore \angle ABP = \angle APB = \theta.$

$\therefore$ $PA = BA$ (Sides opp. equal $\angle$s in a Δ are equal).

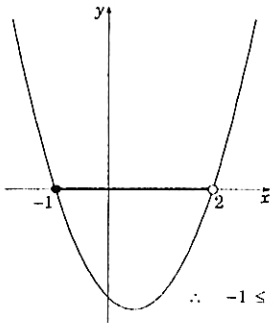
(c) (i) $\sin(\theta + 2\theta)$
$= \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$
$= \sin\theta(1 - 2\sin^2\theta) + \cos\theta \cdot 2\sin\theta\cos\theta$
$= \sin\theta - 2\sin^3\theta + \cos^2\theta \cdot 2\sin\theta$
$= \sin\theta - 2\sin^3\theta + 2\sin\theta(1 - \sin^2\theta)$
$= 3\sin\theta - 4\sin^3\theta.$

(ii) $\sin 3\theta = 2\sin\theta, \quad 0 \le \theta \le 2\pi$
$3\sin\theta - 4\sin^3\theta = 2\sin\theta$
$\sin\theta - 4\sin^3\theta = 0$
$\sin\theta(1 - 4\sin^2\theta) = 0$
$\sin\theta(1 - 2\sin\theta)(1 + 2\sin\theta) = 0.$

$\therefore \sin\theta = 0, \frac{1}{2}, -\frac{1}{2}$
$\theta = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}.$
In ascending order, the solutions are
$\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi.$

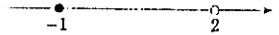
QUESTION 4

(a) $\frac{3x}{x-2} \le 1$
Note $x \ne 2$ since LHS is undefined.
METHOD 1
$\frac{3x}{x-2} - 1 \le 0$
$\frac{2x+2}{x-2} \le 0$
$\frac{2(x+1)}{x-2} \times (x-2)^2 \le 0 \times (x-2)^2$
$2(x+1)(x-2) \le 0 \quad (x \ne 2).$



$\therefore \quad -1 \le x < 2.$

METHOD 2 *Use the critical point method*
Consider $\frac{3x}{x-2} = 1 \quad (x \ne 2)$
$3x = x - 2$
$x = -1.$



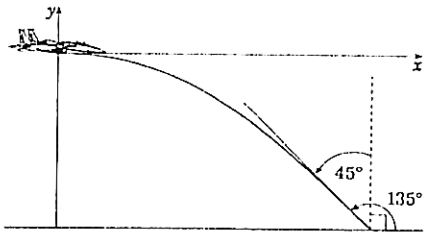
Now consider $\frac{3x}{x-2} \le 1.$

Test $x = -2$: $\frac{-6}{-4} \le 1$ False
$x = 0$: $0 \le 1$ True
$x = 3$: $\frac{9}{1} \le 1$ False

$\therefore \quad -1 \le x < 2.$

METHOD 3
$\frac{3x}{x-2} \le 1.$
If $x - 2 < 0$, ie. $x < 2$,
then $3x \ge x - 2$
$x \ge -1,$
ie. $-1 \le x < 2.$
If $x - 2 > 0$, ie. $x > 2$,
then $3x \le x - 2$
$x \le -1.$
$\therefore$ No further solutions.
$\therefore$ The solution is $-1 \le x < 2.$

(b)



METHOD 1
$x = Vt, \quad y = -5t^2.$
Eliminating the parameter t, $t = \frac{x}{V}.$
$\therefore \quad y = -\frac{5x^2}{V^2}$
$\frac{dy}{dx} = -\frac{10x}{V^2}.$

Note: The angle between the direction of the bomb when it hits the ground and the positive x axis is 135°.

When $x = 4000$, $\frac{dy}{dx} = \tan 135° = -1.$
$\therefore \quad \frac{-10 \times 4000}{V^2} = -1$
$V^2 = 40\,000$
$V = 200.$
$(V > 0$, since it is the speed of the aircraft.)

METHOD 2
$x = Vt \qquad y = -5t^2$
$\dot{x} = V \qquad \dot{y} = -10t$
$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = -\frac{10t}{V}.$
When $x = 4000$, $t = \frac{4000}{V}$ and $\frac{dy}{dx} = -1.$
$\therefore \quad -1 = -\frac{10 \times 4000}{V^2}$
$V^2 = 40\,000$
$\therefore \quad V = 200.$

METHOD 3
An angle of 45° means that the ground is a focal chord. Therefore the plane is flying at an altitude of $4000 \div 2 = 2000$ m.
Hence the bomb hits the ground at time t, given by:
$-2000 = -5t^2 \quad (\text{from } y = -5t^2)$
$t^2 = 400$
$\therefore \quad t = 20.$
Substituting $t = 20$, $x = 4000$
$4000 = 20V \quad (\text{from } x = Vt)$
$\therefore \quad V = 200.$

(c) $\ddot{x} = -4x$
$= -n^2x$
$\therefore \quad n = 2.$
$x = a\cos(nt + \alpha).$
$\dot{x} = -an\sin(nt + \alpha).$
When $t = 0$, $x = 3$
$3 = a\cos\alpha. \quad \text{—①}$
When $t = 0$, $\dot{x} = -6\sqrt{3}$
$-6\sqrt{3} = -2a\sin\alpha$
$a\sin\alpha = 3\sqrt{3}. \quad \text{—②}$
$\tan\alpha = \frac{3\sqrt{3}}{3} = \sqrt{3}$
$\therefore \quad \alpha = \frac{\pi}{3}.$
Substitute in ①: $3 = a\cos\frac{\pi}{3}$
$a = 6$
$\therefore \quad x = 6\cos\left(2t + \frac{\pi}{3}\right).$
Also correct are $x = 6\sin\left(2t + \frac{5\pi}{6}\right)$
and $x = 3\cos 2t - 3\sqrt{3}\sin 2t.$

QUESTION 5

(a) (i) $y = 2\cos^{-1}\frac{x}{3}$
$x = 0, \quad y = 2\cos^{-1}0$
$= 2 \times \frac{\pi}{2}$
$= \pi.$
$\therefore$ The y intercept $= \pi.$

(ii) For the inverse, $x = 2\cos^{-1}\frac{y}{3}$
$\frac{y}{3} = \cos\frac{x}{2}$
$y = 3\cos\frac{x}{2},$
$\therefore \quad f^{-1}(x) = 3\cos\frac{x}{2}.$
Since the range of $y = f(x)$ is $0 \le y \le 2\pi$, the domain of $y = f^{-1}(x)$ is $0 \le x \le 2\pi.$

(iii) $A = \int_0^{\pi} x\,dy$
$= \int_0^{\pi} 3\cos\frac{y}{2}\,dy$
$= 6\left[\sin\frac{y}{2}\right]_0^{\pi}$
$= 6\left(\sin\frac{\pi}{2} - \sin 0\right)$
$= 6.$
$\therefore$ The area is 6 square units.

(b) $(q + p)^n - (q - p)^n$
$= \left[q^n + \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 + \binom{n}{3}q^{n-3}p^3 + \cdots + p^n\right] - \left[q^n - \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 - \binom{n}{3}q^{n-3}p^3 + \cdots + (-1)^n p^n\right]$
$= 2\left[\binom{n}{1}q^{n-1}p + \binom{n}{3}q^{n-3}p^3 + \cdots\right]$
$\therefore \quad (q + p)^n - (q - p)^n$
$= 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3 + \cdots$

If n is odd, $(-1)^n p^n$ is negative,
$\therefore$ last term is $2\binom{n}{n}p^n = 2p^n.$

If n is even, $(-1)^n p^n$ is positive,
$\therefore$ last term is $2\binom{n}{n-1}qp^{n-1} = 2nqp^{n-1}.$

(c) (i) $\left(\frac{5}{6}+\frac{1}{6}\right)^n$ is the binomial probability function, since probability of a six $= \frac{1}{6}$.

$\therefore$ Probability of exactly r sixes,

$$P(r) = \binom{n}{r}\left(\frac{5}{6}\right)^{n-r}\left(\frac{1}{6}\right)^r.$$

(ii) Probability of an odd number of sixes

$$= P(1) + P(3) + P(5) + \cdots$$

$$= \binom{n}{1}\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right) + \binom{n}{3}\left(\frac{5}{6}\right)^{n-3}\left(\frac{1}{6}\right)^3 + \cdots$$

$$= \frac{1}{2}\left[\left(\frac{5}{6}+\frac{1}{6}\right)^n - \left(\frac{5}{6}-\frac{1}{6}\right)^n\right], \quad \text{from (b),}$$

$$= \frac{1}{2}\left[1-\left(\frac{2}{3}\right)^n\right].$$

Note that the expression in (b) doubles up on probabilities for r odd and omits probabilities for r even.

Hence it was applicable to this question.

QUESTION 6

(a) Let $S(n)$ be the statement that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

For $n = 1$, $n^3 + (n+1)^3 + (n+2)^3$
$= 1^3 + 2^3 + 3^3$
$= 36$, which is divisible by 9.

$\therefore$ $S(1)$ is true.

Assume $S(k)$ is true,

$\therefore$ $k^3 + (k+1)^3 + (k+2)^3$
$= 9M$, where M is an integer.

Consider the case when $n = k+1$:

$(k+1)^3 + (k+2)^3 + (k+3)^3$
$= (k+1)^3 + (k+2)^3 + k^3 + 9k^2 + 27k + 27$
$= \left[(k+1)^3 + (k+2)^3 + k^3\right] + 9k^2 + 27k + 27$
$= 9M + 9k^2 + 27k + 27$
$= 9\left(M + k^2 + 3k + 3\right)$, which is divisible by 9, since $M + k^2 + 3k + 3$ is an integer.

$\therefore$ $S(k+1)$ is true when $S(k)$ is true.

But the result is true when $n = 1$, hence it is true when $n = 2$, and so by mathematical induction the result is true for $n = 1, 2, 3, \ldots$

(b) (i) $x^2 = 4ay$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{x}{2a}.$$

At P, $\frac{dy}{dx} = \frac{2at}{2a} = t$.

Gradient of normal $= -\frac{1}{t}$.

Equation of normal is

$$y - at^2 = -\frac{1}{t}(x - 2at)$$
$$ty - at^3 = -x + 2at$$
$$x + ty = at^3 + 2at.$$

(ii) At $Q\left(2aq, aq^2\right)$, gradient of normal is t.

Also at $Q\left(2aq, aq^2\right)$, gradient of normal is $-\frac{1}{q}$.

$\therefore$ $t = -\frac{1}{q}$

$\therefore$ $q = -\frac{1}{t}$

$\therefore$ Q is $\left(-\frac{2a}{t}, \frac{a}{t^2}\right)$.

(iii) Normal at P: $x + ty = at^3 + 2at$ —①

Normal at Q: $x - \frac{1}{t}y = -\frac{a}{t^3} - \frac{2a}{t}$ —②

① $\times t$: $tx + t^2y = at^4 + 2at^2$ —③

② $\times t^3$: $t^3x - t^2y = -a - 2at^2$ —④

③ + ④: $\left(t + t^3\right)x = a\left(t^4 - 1\right)$

$$t\left(t^2+1\right)x = a\left(t^2+1\right)\left(t^2-1\right)$$
$$x = \frac{a\left(t^2-1\right)}{t}$$

$\therefore$ $x = a\left(t - \frac{1}{t}\right)$.

Substitute in ①:

$$a\left(t - \frac{1}{t}\right) + ty = at^3 + 2at$$
$$ty = at^3 + at + \frac{a}{t}$$
$$y = a\left(t^2 + 1 + \frac{1}{t^2}\right).$$

$\therefore$ $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$.

(iv) Parametric equations of the locus of R are

$$\begin{cases} x = a\left(t - \frac{1}{t}\right) & \text{—⑤} \\ y = a\left(t^2 + 1 + \frac{1}{t^2}\right) & \text{—⑥} \end{cases}$$

From ⑤: $x^2 = a^2\left(t^2 - 2 + \frac{1}{t^2}\right)$

$$= a^2\left(t^2 + 1 + \frac{1}{t^2}\right) - 3a^2.$$

$\therefore$ $x^2 = ay - 3a^2$ is the Cartesian equation of the locus.

QUESTION 7

(a) (i) $\frac{dv}{dt} = x - 1$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 1.$$

Integrating, $\frac{1}{2}v^2 = \frac{x^2}{2} - x + c$.

When $x = 0$, $v = 1$:

$\therefore$ $\frac{1}{2} = c$

$\therefore$ $\frac{1}{2}v^2 = \frac{x^2}{2} - x + \frac{1}{2}$

$\therefore$ $v^2 = x^2 - 2x + 1$

$\therefore$ $v^2 = (x-1)^2$.

(ii) $\therefore$ $v = \pm(x-1)$

But if $x = 0$, $v = 1$,

$\therefore$ $v = -(x-1)$

$v = 1 - x$

$\frac{dx}{dt} = 1 - x$

$\frac{dt}{dx} = \frac{1}{1-x}$

$$t = \int \frac{1}{1-x}\,dx$$
$$= -\log_e(1-x) + c_1.$$

When $t = 0$, $x = 0$,

$0 = -\log_e 1 + c_1$

$c_1 = 0$

$\therefore$ $t = -\log_e(1-x)$

$e^{-t} = 1 - x$

$\therefore$ $x = 1 - e^{-t}$.

Checks: $\frac{dx}{dt} = e^{-t} = 1 - x$.

$\frac{dv}{dt} = -e^{-t} = x - 1$.

(b) (i) In ΔAOP,

$AP^2 = OA^2 + OP^2 - 2OA \cdot OP\cos\frac{\pi}{3}$.

From ΔAOT, $AO = h$.

From ΔPOT, $\tan\alpha = \frac{h}{OP}$

$\therefore$ $OP = h\cot\alpha$.

$\therefore$ $AP^2 = h^2 + h^2\cot^2\alpha - 2h \cdot h\cdot\cot\alpha\cdot\frac{1}{2}$

$\therefore$ $AP^2 = h^2 + h^2\cot^2\alpha - h^2\cot\alpha$.

(ii) In ΔATP,

$AP^2 = AT^2 + PT^2 - 2AT \cdot PT\cos\theta$.

In ΔAOT, $AT^2 = h^2 + h^2$

$\therefore$ $AT^2 = 2h^2$.

In ΔPOT, $PT^2 = h^2 + h^2\cot^2\alpha$

$\therefore$ $PT^2 = h^2\operatorname{cosec}^2\alpha$.

$\therefore$ $AP^2 = 2h^2 + h^2 + h^2\cot^2\alpha - 2\sqrt{2}\,h \cdot h\operatorname{cosec}\alpha\cdot\cos\theta$,

$\therefore$ $AP^2 = 3h^2 + h^2\cot^2\alpha - 2\sqrt{2}h^2\operatorname{cosec}\alpha\cdot\cos\theta$.

Equating expressions for AP^2,

$h^2 + h^2\cot^2\alpha - h^2\cot\alpha$
$= 3h^2 + h^2\cot^2\alpha - 2\sqrt{2}\,h^2\operatorname{cosec}\alpha\cdot\cos\theta$.

$2\sqrt{2}h^2\operatorname{cosec}\alpha\cos\theta = 2h^2 + h^2\cot\alpha$ —①

① $\times \frac{\sin\alpha}{h^2}$:

$2\sqrt{2}\cos\theta = 2\sin\alpha + \cos\alpha$

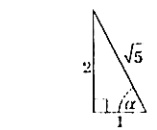
$\therefore$ $\cos\theta = \frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha$.

(iii) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)$

$$\frac{d\theta}{d\alpha} = -\frac{-\left(\frac{1}{\sqrt{2}}\cos\alpha - \frac{1}{2\sqrt{2}}\sin\alpha\right)}{\sqrt{1-\left(\frac{1}{\sqrt{2}}\sin\alpha + \frac{1}{2\sqrt{2}}\cos\alpha\right)^2}}.$$

At stationary points, $\frac{d\theta}{d\alpha} = 0$.

$$\frac{1}{2\sqrt{2}}\sin\alpha - \frac{1}{\sqrt{2}}\cos\alpha = 0$$
$$\sin\alpha - 2\cos\alpha = 0$$
$$\tan\alpha = 2$$
$$\alpha = \tan^{-1}2.$$



$\alpha \doteqdot 1{\cdot}107$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\cdot\frac{2}{\sqrt{5}} + \frac{1}{2\sqrt{2}}\cdot\frac{1}{\sqrt{5}}\right)$$
$$= \cos^{-1}\left(\frac{\sqrt{10}}{5} + \frac{\sqrt{10}}{20}\right)$$
$$= \cos^{-1}\frac{\sqrt{10}}{4}.$$

Stationary point at $\left(\tan^{-1}2,\ \cos^{-1}\frac{\sqrt{10}}{4}\right)$.

If $\alpha < \tan^{-1}2$, $\frac{d\theta}{d\alpha} < 0$

If $\alpha > \tan^{-1}2$, $\frac{d\theta}{d\alpha} > 0$

for $0 < \alpha < \frac{\pi}{2}$.

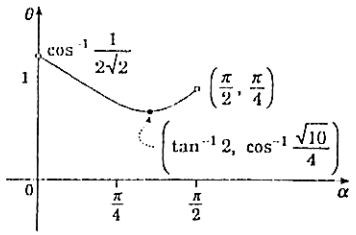
$\therefore$ Minimum turning point at $\left(\tan^{-1}2,\ \cos^{-1}\frac{\sqrt{10}}{4}\right)$.

If $\alpha = 0$, $\theta = \cos^{-1}\dfrac{1}{2\sqrt{2}}$ $(\doteqdot 1{\cdot}209)$.

If $\alpha = \dfrac{\pi}{2}$, $\theta = \cos^{-1}\dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4}$.

As $\alpha \to 0$, $\theta \to \cos^{-1}\dfrac{1}{2\sqrt{2}}$.

As $\alpha \to \dfrac{\pi}{2}$, $\theta \to \dfrac{\pi}{4}$.



In an alternative approach to this question, we can write $\dfrac{1}{\sqrt{2}}\sin\alpha + \dfrac{1}{2\sqrt{2}}\cos\alpha$ in the form $a\cos(\alpha - \phi)$, leading to

$$\theta = \cos^{-1}\left[\frac{\sqrt{10}}{4}\cos\left(\alpha - \tan^{-1}2\right)\right].$$

θ then has a minimum when

$\cos\left(\alpha - \tan^{-1}2\right)$ has a maximum.

This occurs when

$$\cos\left(\alpha - \tan^{-1}2\right) = 1$$
$$\alpha = \tan^{-1}2$$
$$\theta = \cos^{-1}\frac{\sqrt{10}}{4}.$$

END OF MATHEMATICS EXTENSION 1 SOLUTIONS