CSSA

2025
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

DO NOT REMOVE PAPER FROM EXAMINATION ROOM

Centre Number

Student Number

Mathematics Extension 1

Afternoon Session
Friday, 15 August 2025

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- A normal cumulative distribution function table is provided
- Use the Multiple-Choice Answer Sheet provided for Section I
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–11)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet for Questions 1–10

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1 What is the coefficient of x in the expansion of $\left(\frac{2}{x}-x\right)^5$?

A. -40

B. -10

C. 10

D. 40

2 Given $Q = 10+4e^{2t}$, which of the following is the rate of change of Q?

A. $\frac{dQ}{dt} = (Q-5)$

B. $\frac{dQ}{dt} = (Q-10)$

C. $\frac{dQ}{dt} = 2(Q-5)$

D. $\frac{dQ}{dt} = 2(Q-10)$

3 In how many different ways can all the letters of the word "SCINTILLA" be arranged?

A. 1260

B. 5040

C. 90720

D. 181440

4 Which of the following is the correct factorisation of $P(x) = x^3 + 4x^2 + x - 6$?

A. $P(x) = (x+1)(x+2)(x-3)$

B. $P(x) = (x+1)(x-2)(x+3)$

C. $P(x) = (x-1)(x+2)(x+3)$

D. $P(x) = (x-1)(x-2)(x-3)$

5 For the function $y = \sin 2x \cos 2x$, which of the following is the correct amplitude and period?

A. Amplitude is $\frac{1}{2}$ and period is π

B. Amplitude is $\frac{1}{2}$ and period is $\frac{\pi}{2}$

C. Amplitude is 1 and period is π

D. Amplitude is 1 and period is $\frac{\pi}{2}$

6 Which of the following functions is a constant function?

A. $y = \arccos(x) + \arcsin(-x)$

B. $y = \arccos(x) - \arcsin(-x)$

C. $y = \arcsin(x) - \arccos(x)$

D. $y = \arcsin(x) + \arccos(-x)$

7 Suppose 12 firecrackers and 8 sparklers are to be arranged in a row so that no two sparklers are together.

How many ways can they be arranged?

A. 165

B. 495

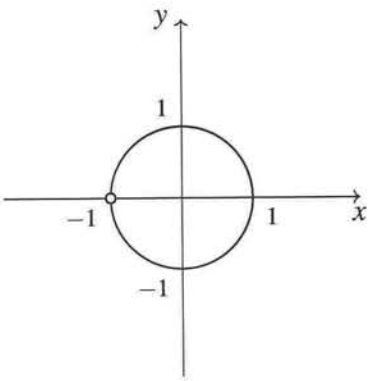
C. 792

D. 1287

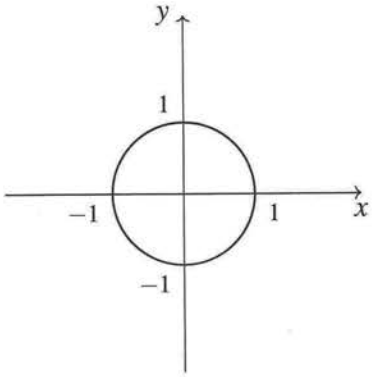
8 A curve is defined parametrically as $\begin{cases} x = \frac{1-t^2}{1+t^2} \\ y = \frac{2t}{1+t^2} \end{cases}$ for $t \in [0, \infty)$.

Which of the following graphs best represents this curve?

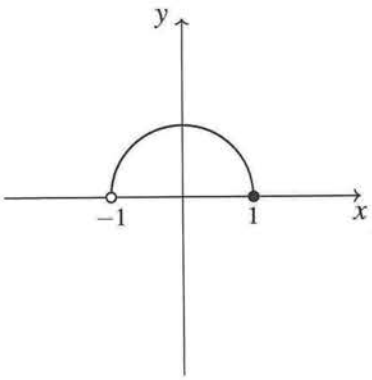
A.



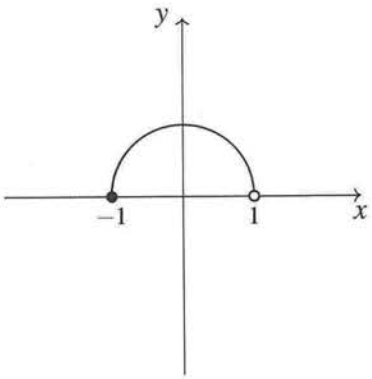
B.



C.



D.



9 Which of the statements is **always** true for any non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

A. $|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}| \leq |\underset{\sim}{a}|$

B. $|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}| \geq |\underset{\sim}{a}|$

C. $|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}| \leq |\underset{\sim}{b}|$

D. $|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}| \geq |\underset{\sim}{b}|$

10 The polynomial $P(x) = x^3 + px - 2$ has three distinct, real, non-zero roots.

Which of the following statements must **always** be true for p?

A. $p < -3$

B. $p > -3$

C. $p < 3$

D. $p > 3$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Your responses for Questions 11–14 should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks)

(a) Consider the vectors $\underset{\sim}{u} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$. **2**

Calculate the value of $|\underset{\sim}{u} + \underset{\sim}{v}|$.

(b) Find $\int_0^2 \frac{1}{4+x^2}\,dx$. **2**

(c) Use the substitution $u = \sqrt{x}$ to find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$. **2**

(d) Five fair dice, each with faces labelled 1 to 6, are rolled. Calculate the probability that at least two dice land with a 3 uppermost. **2**

(e) Solve $\frac{1}{x-2} \geq 3$. **3**

(f) Find a unit vector perpendicular to the vector $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$. **2**

(g) The proportion of Australian residents that live in rural or remote regions is 28%. **3**
A random sample of 200 residents is chosen from the entire Australian population.

Use the standard normal distribution and the information on page 13 to estimate the probability that, within this sample, there are less than 60 rural or regional residents.

Question 12 (14 marks)

(a) Find $\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{x}{2}\right) dx$. **3**

(b) Solve $\sqrt{3}\sin x - \cos x = 2$ for $0 \leq x \leq 2\pi$. **3**

(c) Use mathematical induction to prove that **3**

$$1+6+15+\cdots+(2n^2-n) = \frac{1}{6}n(n+1)(4n-1)$$

for integers $n \geq 1$.

(d) The position vector of a projectile, fired from the origin with velocity V m/s, after t seconds is given by **3**

$$\underset{\sim}{r}(t) = \frac{V}{2}t\,\underset{\sim}{i} + \left(\frac{\sqrt{3}V}{2}t - 5t^2\right)\underset{\sim}{j}.$$

Find the minimum velocity required for the projectile to reach a height of 40 metres. Round your answer to the nearest m/s.

(e) Each of the students in a class writes a different two-digit number on the blackboard. The teacher claims that "there are at least three numbers on the board the sum of whose digits are the same". **2**

Determine the smallest number of students in the class for the teacher's claim to be correct.

Question 13 (15 marks)

(a) Given $f(x) = 1 + 2e^{-x}$, $x \geq 0$, sketch the graph of $g(x) = \dfrac{1}{f(x)}$, clearly marking any intercepts and asymptotes. **2**

(b) Planes are scheduled for take-off from the main runway at Sydney Airport on a bearing of 340° T from 8:00 pm. At the same time, a weather system is producing 15 ms^{-1} winds that change direction as the system moves across the airport.

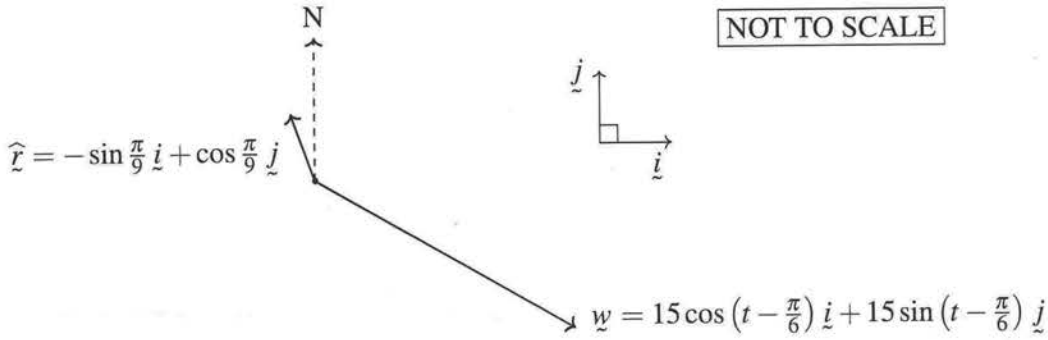
The wind vector at time t hours after 8:00 pm is given by

$$\underset{\sim}{w} = 15\cos\left(t - \frac{\pi}{6}\right)\underset{\sim}{i} + 15\sin\left(t - \frac{\pi}{6}\right)\underset{\sim}{j}$$

and the unit vector in the direction of take-off is

$$\hat{\underset{\sim}{r}} = -\sin\frac{\pi}{9}\underset{\sim}{i} + \cos\frac{\pi}{9}\underset{\sim}{j}.$$

This situation at 8:00 pm is represented in the diagram below, where the standard basis vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ point east and north, respectively.



(i) Show that $\underset{\sim}{w} \cdot \hat{\underset{\sim}{r}} = 15\sin\left(t - \dfrac{5\pi}{18}\right)$. **2**

(ii) Runway use is determined by wind strength and direction. Once the tailwind (the component of the wind in the same direction as take-off) exceeds 5 ms^{-1}, Air Traffic Control will order a change in runway direction. Determine the time, to the nearest minute, when the order to change the direction of take-off is issued. **2**

Question 13 continues on page 9

(c) A rare species of butterfly inhabits a national park. The growth rate of the butterfly population varies according to the time of year and the number of butterflies present. An entomologist studying the butterflies suggests the following differential equation to describe the population growth **4**

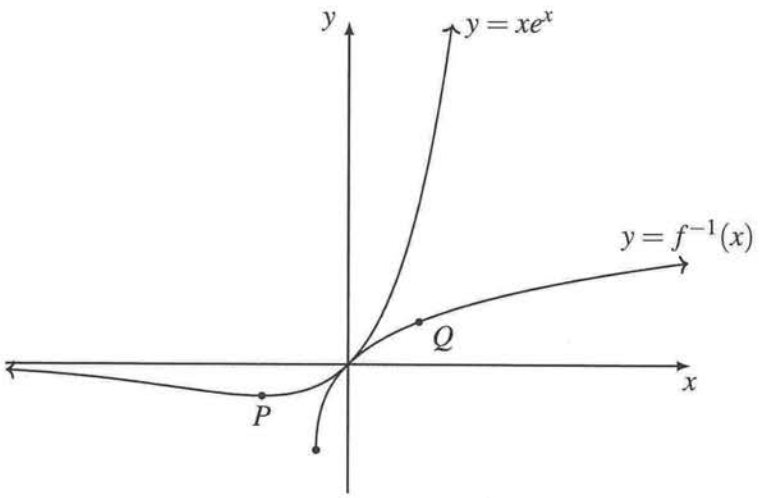
$$\frac{dn}{dt} = \frac{\pi n}{6}\cos\left(\frac{\pi t}{6}\right),$$

where n is number of butterflies (in 100 000s) and t is the time in months after 1 January.

The entomologist estimates that there are 100000 butterflies in the national park at the start of the year.

Find the maximum and minimum number of butterflies in the national park over the course of a year. Write your answer correct to the nearest 1000 butterflies.

(d) The diagram below shows the graph of the function $y = xe^x$, which has a global minimum at P. The diagram also shows the graph of $y = f^{-1}(x)$, the inverse function of $y = xe^x$, which includes the positive x-axis in its domain. The point Q lies on $y = f^{-1}(x)$.



(i) Determine the coordinates of P. Write your answer in exact form. **2**

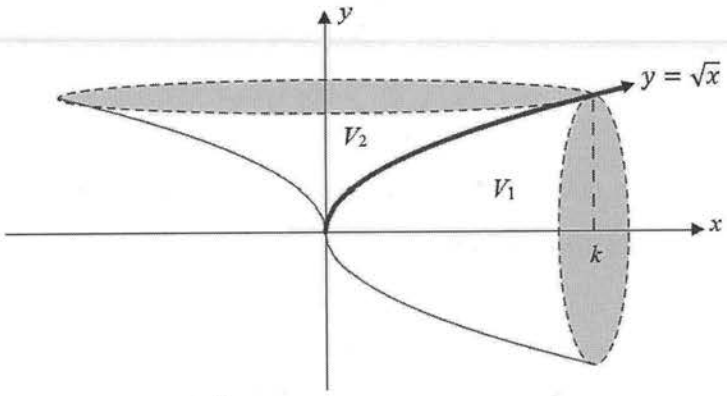
(ii) State the largest possible domain of $f^{-1}(x)$. **1**

(iii) Find the equation of the tangent to $y = f^{-1}(x)$ at $Q\left(\frac{\sqrt{e}}{2}, \frac{1}{2}\right)$. **2**

End of Question 13

Question 14 (15 marks)

(a) The arc of the curve $y = \sqrt{x}$ from $x = 0$ to $x = k$ is rotated around the x-axis to form a solid with volume V_1 and also around the y-axis to form a solid with volume V_2 as shown in the diagram below. **3**



For which value of k is $V_1 = V_2$?

(b) Louis is playing a poker machine at his local club. Each game on the poker machine is a Bernoulli trial where the probability of winning is p, with $0 < p < 1$. Louis is determined to keep playing until he wins one game.

The random variable X is the total number of games that Louis plays. Thus

$$\begin{aligned} P(X = 1) &= p, \\ P(X = 2) &= p(1-p), \\ P(X = 3) &= p(1-p)^2, \\ &\vdots \\ P(X = n) &= p(1-p)^{n-1}, \end{aligned}$$

where n is a positive integer.

(i) Show that the probability that Louis plays more than n games is $(1-p)^n$. **2**

(ii) If m and n are two positive integers, show that **2**

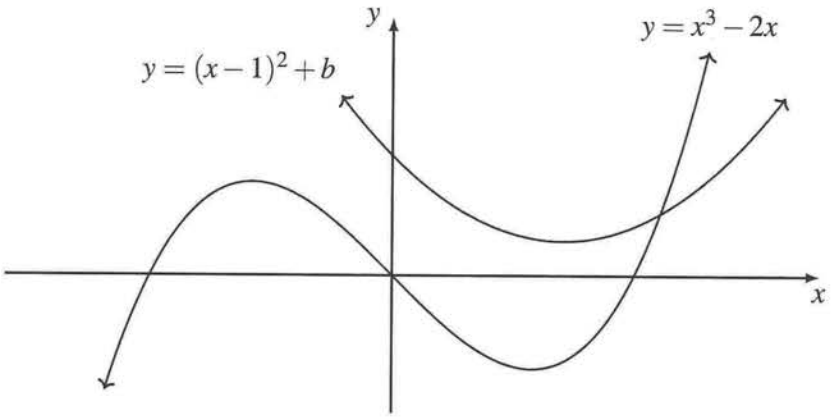
$$P(X > m+n \mid X > m) = P(X > n).$$

(iii) Louis thinks that he is "building up luck" by playing the same poker machine, even when he loses. Explain why this notion is incorrect. **1**

Question 14 continues on page 11

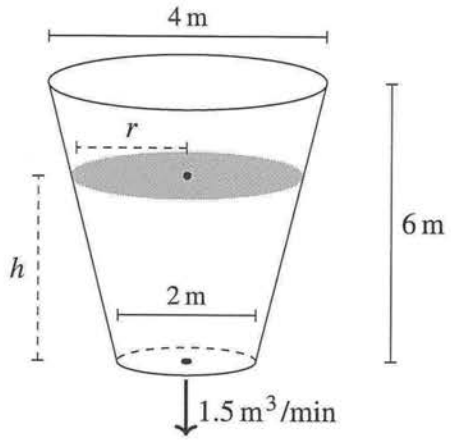
Question 14 (continued)

(c) Consider the functions $y = x^3 - 2x$ and $y = (x-1)^2 + b$ as shown in the diagram below. **3**



For which two values of b do the graphs intersect exactly twice?

(d) A tank in the shape of a truncated cone is filled with water, which is leaking out from the bottom at a rate of 1.5 m^3 per minute. **4**



The volume V, in cubic metres, of water in this tank when the height of the water is h metres and the radius of the surface of the water is r metres is given by

$$V = \frac{\pi}{3} h\left(r^2 + r + 1\right). \qquad \text{(Do NOT prove this.)}$$

Find the rate at which the water level is falling when the height of the remaining water is 1 metre. Give your answer correct to the nearest centimetre per minute.

End of Examination

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NORMAL CUMULATIVE DISTRIBUTION FUNCTION

Entries represent $P(Z \leqslant z)$. The value of z to the first decimal place is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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2025

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MARKING GUIDELINES

Mathematics Extension I

Section I
10 marks

Multiple Choice Answer Key

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	A	ME11-5	E1
2	D	ME11-4	E1
3	C	ME11-5	E1-E2
4	C	ME11-2	E2
5	B	ME11-3	E2
6	B	ME11-3, ME12-4	E3
7	D	ME11-5	E3
8	C	ME11-2	E4
9	A	ME12-2	E4
10	A	ME11-2	E4

Question 1 (1 mark)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E1*

Solution	Mark
$\left(\frac{2}{x}-x\right)^5 = \sum_{k=0}^{5}\binom{5}{k}\left(\frac{2}{x}\right)^{5-k}(-x)^k$ $= \sum_{k=0}^{5}\binom{5}{k}(-1)^k \times 2^{5-k} \times x^{k-5} \times x^k$ $= \sum_{k=0}^{5}\binom{5}{k}(-1)^k \times 2^{5-k} \times x^{2k-5}$ We want the term in x, i.e. when $2k-5=1$, i.e. when $k=3$. So the coefficient is $\binom{5}{3}(-1)^3 \times 2^2 = -40$. Hence A	1

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Question 2 (1 mark)
Outcomes Assessed: *ME11-4*
Targeted Performance Bands: *E1*

Solution	Mark
$Q = 10 + 4e^{2t} \Rightarrow Q - 10 = 4e^{2t}$ $\frac{dQ}{dt} = 2 \times 4e^{2t}$ $= 2(Q - 10)$ Hence D	1

Question 3 (1 mark)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E1-E2*

Solution	Mark
There are nine letters altogether, of which 2 are repeated. Thus the answer is $\frac{9!}{2! \times 2!} = 90720.$ Hence C	1

Question 4 (1 mark)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E2*

Solution	Mark
Consider $P(x) = x^3 + 4x^2 + x - 6$. Test $P(\alpha)$ such that α is a factor of -6. Since $P(x)$ is a polynomial of degree 3, there are at most 3 linear factors. $P(1) = 0$ so $x - 1$ is a factor of $P(x)$ $P(-2) = 0$ so $x + 2$ is a factor of $P(x)$ $P(-3) = 0$ so $x + 3$ is a factor of $P(x)$ Thus $P(x) = (x-1)(x+2)(x+3)$. Hence C	1

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Question 5 (1 mark)
Outcomes Assessed: *ME11-3*
Targeted Performance Bands: *E2*

Solution	Mark
$y = \sin 2x \cos 2x = \frac{1}{2}\sin 4x$ $\therefore$ amplitude is $\frac{1}{2}$ period is $\frac{\pi}{2}$ Hence B	1

Question 6 (1 mark)
Outcomes Assessed: *ME11-3, ME12-4*
Targeted Performance Bands: *E3*

Solution	Mark
If the derivative of a function is zero, then the function is a constant function. Consider the derivative of each of the functions. A: $y = \arccos(x) + \arcsin(-x) \Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}} \neq 0$ B: $y = \arccos(x) - \arcsin(-x) \Rightarrow \frac{dy}{dx} = 0$ C: $y = \arcsin(x) - \arccos(x) \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}} \neq 0$ D: $y = \arcsin(x) + \arccos(-x) \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}} \neq 0$ Hence B	1

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Question 7 (1 mark)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E3*

Solution	Mark
Consider the 12 firecrackers arranged in a row: FFFFFFFFFFFF There are 13 gaps in which a sparkler may be placed ensuring no two sparklers are next to one another (including before the first F and after the last F). From these gaps, 8 must be chosen in which to place a sparkler. Thus the solution is ${}^{13}C_8 = \dfrac{13!}{8! \times 5!} = 1287$. Hence D	1

Question 8 (1 mark)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E4*

Solution	Mark
Note that $y \geqslant 0$ for $t \in [0, \infty)$, which eliminates A and B. Also note that when $t = 1$, $x = 0$, $y = 1$, thus D is eliminated. Thus the correct answer is C. Hence C	1

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Question 9 (1 mark)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E4*

Solution	Mark
Consider the following: $\|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\| = \left\| \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\|\underset{\sim}{b}\|}\,\widehat{\underset{\sim}{b}} \right\|$ $= \dfrac{\|\underset{\sim}{a}\cdot\underset{\sim}{b}\|}{\|\underset{\sim}{b}\|}\,\|\widehat{\underset{\sim}{b}}\|$ $= \dfrac{\|\underset{\sim}{a}\|\,\|\underset{\sim}{b}\|\,\|\cos\theta\|}{\|\underset{\sim}{b}\|}$, since $\|\widehat{\underset{\sim}{b}}\| = 1$ $= \|\underset{\sim}{a}\|\,\|\cos\theta\|$ Now, $0 \le \|\cos\theta\| \le 1$ thus $0 \le \|\underset{\sim}{a}\|\,\|\cos\theta\| \le \|\underset{\sim}{a}\|$ and so $\|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\| \le \|\underset{\sim}{a}\|$ is **always** true for any non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Hence A	1

Question 10 (1 mark)
Outcomes Assessed: *ME11-2*
Targeted Performance Bands: *E4*

Solution	Mark
Consider $p = 0$, $P(x) = x^3 - 2 = \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{2}^2\right)$. Now, $x^2 + \sqrt[3]{2}x + \sqrt[3]{2}^2 = 0$ has no real solutions, meaning $P(x) = x^3 - 2$ will have only one distinct zero and hence $p \neq 0$. This eliminates solutions B and C. Now consider, $P'(x) = 3x^2 + p$. For $P(x) = 0$ to have three distinct, real, non-zero roots, it must have exactly two turning points and as such, $P'(x) = 0$ must have exactly two distinct solutions. This eliminates option D since $3x^2 + p = 0$ has no real solutions if $p > 3$. Note: If $p = -3$ there is a double root at $x = 1$, so p is strictly less than -3. Hence A	1

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Section II

60 marks

Question 11 (16 marks)

Question 11(a) (2 marks)

Outcomes Assessed: *ME12-2*

Targeted Performance Bands: *E1*

Criteria	Marks
• correct solution	2
• correctly finds $\underset{\sim}{u}+\underset{\sim}{v}$	1

Sample Answer:

$$\begin{aligned}\underset{\sim}{u}+\underset{\sim}{v} &= \begin{pmatrix}5\\12\end{pmatrix}+\begin{pmatrix}3\\3\end{pmatrix}\\ &= \begin{pmatrix}8\\15\end{pmatrix}\\ |\underset{\sim}{u}+\underset{\sim}{v}| &= \left|\begin{pmatrix}8\\15\end{pmatrix}\right|\\ &= \sqrt{8^2+15^2}\\ &= 17 \text{ units}\end{aligned}$$

Question 11(b) (2 marks)

Outcomes Assessed: *ME12-1, ME12-4*

Targeted Performance Bands: *E1*

Criteria	Marks
• provides correct solution	2
• finds the correct indefinite integral	1

Sample Answer:

$$\begin{aligned}\int_0^2 \frac{1}{4+x^2}\,dx &= \left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2\\ &= \frac{1}{2}\left[\tan^{-1}\left(\frac{2}{2}\right)-\tan^{-1}0\right]\\ &= \frac{1}{2}\left[\frac{\pi}{4}-0\right]\\ &= \frac{\pi}{8}\end{aligned}$$

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Question 11(c) (2 marks)
Outcomes Assessed: *ME12-1, ME12-4*
Targeted Performance Bands: *E1-E2*

Criteria	Marks
• correct solution	2
• valid attempt at substitution	1

Sample Answer:
Using the substitution $u = \sqrt{x}$ then $x = u^2$.

$$\begin{aligned} x &= u^2 \\ \frac{dx}{du} &= 2u \\ dx &= 2u\,du \end{aligned}$$

Substituting gives
$$\begin{aligned} \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx &= \int \frac{e^u}{u}\,2u\,du \\ &= \int 2e^u\,du \\ &= 2e^u + c \\ &= 2e^{\sqrt{x}} + c \end{aligned}$$

Question 11(d) (2 marks)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E2*

Criteria	Marks
• correct solution	2
• calculates the probability of the complementary event	1

Sample Answer:
The event that at least two dice land with a 3 uppermost is the complement of the event that either zero dice or one die lands with a 3 uppermost.
Let E be the event "at least two dice land with a 3 uppermost"

$$\begin{aligned} P(E) &= 1 - {}^5C_5 \times \left(\frac{5}{6}\right)^5 - {}^5C_4 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^4 \\ &= \frac{763}{3888} \\ &\approx 0.1962 \end{aligned}$$

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Question 11(e) (3 marks)
Outcomes Assessed: *ME11-2, ME11-7*
Targeted Performance Bands: *E2*

Criteria	Marks
• correct solution	3
• valid progress towards the solution such as correct x values with incorrect inequality symbol(s)	2
• attempts a valid method for solving inequalities with an unknown in the denominator	1

Sample Answer:

$$\frac{1}{x-2} \geq 3 \quad x \neq 2$$
$$\frac{1}{x-2} - 3 \geq 0$$
$$\frac{1-3(x-2)}{x-2} \geq 0$$
$$\frac{7-3x}{x-2} \geq 0$$
$$(7-3x)(x-2) \geq 0 \quad \text{multiplying by } (x-2)^2$$
$$\therefore \quad 2 < x \leq \frac{7}{3}, \quad (\text{since } x \neq 2)$$

Question 11(f) (2 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E2*

Criteria	Marks
• provides correct solution	2
• valid progress such as finding the gradient of the vector perpendicular to $\mathbf{a}$ or finding any vector perpendicular, e.g. $\mathbf{a}_\perp = 2\mathbf{i} - \mathbf{j}$ or $\mathbf{a}_\perp = -2\mathbf{i} + \mathbf{j}$	1

Sample Answer:
The vector $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$ has gradient $m = 2$.

Hence a vector perpendicular to $\mathbf{a}$ has gradient $m_\perp = -\frac{1}{2}$.

There are two such vectors with gradient $-\frac{1}{2}$: $2\mathbf{i} - \mathbf{j}$ or $-2\mathbf{i} + \mathbf{j}$.

Both of these vectors have magnitude $\sqrt{5}$.

Hence a unit vector perpendicular to $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$ is $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$ or $-\frac{2}{\sqrt{5}}\mathbf{i} + \frac{1}{\sqrt{5}}\mathbf{j}$.

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Question 11(g) (3 marks)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E2*

Criteria	Marks
• provides correct solution	3
• calculates the z-score for $\hat{p}$	2
• calculates the standard deviation of the sample	1

Sample Answer:
The normal approximation to $\hat{p}$ has mean, $\mu = 0.28$

$$\text{and standard deviation } \sigma = \sqrt{\frac{0.28 \times 0.72}{200}}.$$

We need to find $P(\hat{p} < 0.3)$, so we find the z-score.

$$z = \frac{0.3 - 0.28}{\sigma} \approx 0.63 \quad \text{(correct to 2 significant figures)}$$

$$\therefore \quad P(\hat{p} < 0.3) = P(z < 0.63)$$
$$\approx 0.7357$$

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Question 12 (14 marks)

Question 12(a) (3 marks)

Outcomes Assessed: *ME12-1, ME12-4*

Targeted Performance Bands: *E2*

Criteria	Marks
• correct solution	3
• finding the correct indefinite integral	2
• correct manipulation of the expression $\cos^2\left(\frac{x}{2}\right)$	1

Sample Answer:

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{x}{2}\right) dx &= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(1+\cos\left(2\times\frac{1}{2}x\right)\right) dx \\
&= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos x)\, dx \\
&= \frac{1}{2}\Big[x+\sin x\Big]_0^{\frac{\pi}{2}} \\
&= \frac{1}{2}\left[\frac{\pi}{2}+\sin\frac{\pi}{2}\right]-\frac{1}{2}\Big[0+\sin 0\Big] \\
&= \frac{\pi}{4}+\frac{1}{2} \\
&= \frac{\pi+2}{4}
\end{aligned}$$

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Question 12(b) (3 marks)
Outcomes Assessed: *ME12-3*
Targeted Performance Bands: *E2*

Criteria	Marks
• correct solution	3
• substantial progress such as finding both R and α	2
• valid attempt at using the auxiliary angle method such as finding R or α or using t-formulae substitution	1

Sample Answer:
Let $\sqrt{3}\sin x - \cos x = R\sin(x-\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Then $\sqrt{3}\sin x - \cos x = R\sin x\cos\alpha - R\sin\alpha\cos x$.
Equating coefficients: $R\cos\alpha = \sqrt{3}$ and $R\sin\alpha = 1$.

$$R^2\cos^2\alpha + R^2\sin^2\alpha = 3+1$$
$$R^2 = 4$$
$$\text{so} \quad R = 2, \text{ since } R > 0.$$
$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$$
$$\tan\alpha = \frac{1}{\sqrt{3}}$$
$$\alpha = \frac{\pi}{6}, \text{ since } 0 < \alpha < \frac{\pi}{2}$$
$$\text{So} \quad \sqrt{3}\sin x - \cos x = 2\sin\left(x - \frac{\pi}{6}\right) = 2$$
$$\sin\left(x - \frac{\pi}{6}\right) = 1$$
$$x - \frac{\pi}{6} = \frac{\pi}{2}$$
$$x = \frac{2\pi}{3}$$

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Question 12(c) (3 marks)
Outcomes Assessed: *ME12-1, ME12-7*
Targeted Performance Bands: *E2*

Criteria	Marks
• provides full and correct proof	3
• correctly substitutes the result from the inductive hypothesis	2
• proves the result true for $n = 1$	1

Sample Answer:

Step 1 Prove the result true for $n = 1$.

$$\text{LHS} = 1 \qquad \text{RHS} = \frac{1}{6} \times 1 \times (1+1) \times (4-1)$$
$$= 1$$
$$= \text{LHS}$$

Since LHS = RHS, the result is true for $n = 1$.

Step 2 Let the result be true for some integer $n = k$.

That is, $1 + 6 + 15 + \cdots + (2k^2 - k) = \frac{1}{6}k(k+1)(4k-1)$

Step 3 Prove the result true for $n = k+1$

i.e. Prove that

$$1 + 6 + 15 + \cdots + (2k^2 - k) + \left(2(k+1)^2 - (k+1)\right) = \frac{1}{6}(k+1)(k+2)(4k+3)$$

$$\begin{aligned}
\text{LHS} &= 1 + 6 + 15 + \cdots + (2k^2 - k) + \left(2(k+1)^2 - (k+1)\right) \\
&= \frac{1}{6}k(k+1)(4k-1) + 2(k+1)^2 - (k+1) \qquad \text{By Step 2} \\
&= \frac{1}{6}k(k+1)(4k-1) + \frac{12}{6}(k+1)^2 - \frac{6}{6}(k+1) \\
&= \frac{1}{6}(k+1)\left[k(4k-1) + 12(k+1) - 6\right] \\
&= \frac{1}{6}(k+1)\left[4k^2 + 11k + 6\right] \\
&= \frac{1}{6}(k+1)(k+2)(4k+3) \\
&= \text{RHS as required}
\end{aligned}$$

Hence $1 + 6 + 15 + \cdots + (2n^2 - n) = \frac{1}{6}n(n+1)(4n-1)$ is true for all integers $n \geq 1$ by the principle of mathematical induction.

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Question 12(d) (3 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	3
• correctly finds the expression for t for which the vertical velocity is zero	2
• differentiates to find the velocity of the projectile	1

Sample Answer:
We need the velocity in the vertical direction to equal zero when the vertical position of the projectile is 40 m.

$$r_y = \frac{\sqrt{3}V}{2}t - 5t^2$$
$$v_y = \frac{\sqrt{3}V}{2} - 10t$$

Solving $v_y = 0$

$$\frac{\sqrt{3}V}{2} - 10t = 0$$
$$10t = \frac{\sqrt{3}V}{2}$$
$$t = \frac{\sqrt{3}V}{20}$$

Substituting $t = \frac{\sqrt{3}V}{20}$ into r_y and solving $r_y = 40$:

$$\frac{\sqrt{3}V}{2} \times \frac{\sqrt{3}V}{20} - 5\left(\frac{\sqrt{3}V}{20}\right)^2 = 40$$
$$\frac{3V^2}{40} - \frac{3V^2}{80} = 40$$
$$\frac{3V^2}{80} = 40$$
$$3V^2 = 3200$$
$$V^2 = \frac{3200}{3}$$
$$V = \pm 32.659\cdots$$
$$V \approx 33 \text{ m/s (nearest m/s)}, \ V > 0$$

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Question 12(e) (2 marks)
Outcomes Assessed: *ME11-5*
Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	2
• some progress towards solution such as finding the number 'pigeon-holes' (18) or equivalent	1

Sample Answer:
The first digit can be 1 to 9 inclusive and the second digit can be 0 to 9 inclusive. This gives 18 possible sums of the digits of the two-digit numbers.
Note that every student writes a different two-digit number on the board.
However, the sums of 1 and 18 can only be obtained from one two-digit number each (10 and 99 respectively).
Similarly, the sums of 2 and 17 can only be obtained from two two-digit numbers each (11, 20 and 89 and 98 respectively).
All other sums can be obtained in at least three ways.
Consider the "pigeon-holes" to be the numbers 3 to 16 (a total of 14 sums that may be obtained in three ways so may fit 3 "pigeons") and the pigeons to be the sum of the two-digit number written on the board.
There are also 6 extra "pigeon-holes" that may fit ONLY one pigeon in them - these are for the two-digit numbers 10, 11, 20, 89, 98 and 99.
The smallest number of students will be $6 + 2 \times 14 + 1 = 35$.

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Question 13 (15 marks)

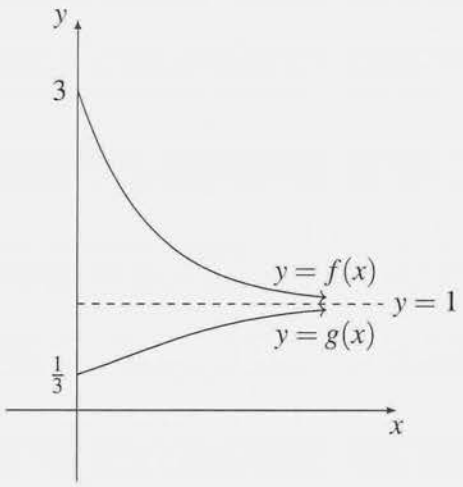
Question 13(a) (2 marks)

Outcomes Assessed: *ME11-1, ME11-2, ME12-1*

Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct graph with correct shape, y-intercept and asymptote clearly marked	2
• one of correct shape, y-intercept or asymptote of $y = g(x)$ OR correct sketch of $f(x)$	1

Sample Answer:



Question 13(b) (i) (2 marks)

Outcomes Assessed: *ME12-2*

Targeted Performance Bands: *E3*

Criteria	Marks
• correct and clear progress to the solution	2
• correctly sets up $\underset{\sim}{w} \cdot \hat{\underset{\sim}{r}}$	1

Sample Answer:

$$\begin{aligned}
\underset{\sim}{w} \cdot \hat{\underset{\sim}{r}} &= 15\cos\left(t - \frac{\pi}{6}\right) \times -\sin\frac{\pi}{9} + 15\sin\left(t - \frac{\pi}{6}\right) \times \cos\frac{\pi}{9} \\
&= 15\left[\sin\left(t - \frac{\pi}{6}\right)\cos\frac{\pi}{9} - \cos\left(t - \frac{\pi}{6}\right)\sin\frac{\pi}{9}\right] \\
&= 15\sin\left(t - \frac{\pi}{6} - \frac{\pi}{9}\right) \\
&= 15\sin\left(t - \frac{5\pi}{18}\right), \text{ as required.}
\end{aligned}$$

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Question 13(b) (ii) (2 marks)
Outcomes Assessed: *ME12-2*
Targeted Performance Bands: *E3*

Criteria	Marks
• correct solution	2
• some progress	1

Sample Answer:
As $\underset{\sim}{r}$ is a unit vector, the component of $\underset{\sim}{w}$ along the runway is given by $(\underset{\sim}{w} \cdot \hat{\underset{\sim}{r}})\hat{\underset{\sim}{r}}$.

To determine when the order to change directions is issued, we solve $\underset{\sim}{w} \cdot \hat{\underset{\sim}{r}} = 5$ for t.

$$\begin{aligned} 15\sin\left(t - \frac{5\pi}{18}\right) &= 5 \\ \sin\left(t - \frac{5\pi}{18}\right) &= \frac{1}{3} \\ \therefore \quad t &= \frac{5\pi}{18} + \sin^{-1}\left(\frac{1}{3}\right) \\ &\approx 1.21250\ldots \\ &= 1\text{ h }13\text{ min (to the nearest minute)} \end{aligned}$$

The order to change runway direction will be issued at 9:13 pm.

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Question 13(c) (4 marks)
Outcomes Assessed: *ME12-1, ME12-4, ME12-7*
Targeted Performance Bands: *E3*

Criteria	Marks
• correctly calculates the maximum and minimum populations	4
• identifies when the maximum and minimum values will occur	3
• uses the initial conditions to find either the constant of integration C or A	2
• progress towards solving the differential equation by separation of variables	1

Sample Answer:

$$\int \frac{1}{n}\,dn = \int \frac{\pi}{6}\cos\left(\frac{\pi t}{6}\right)\,dt$$

$$\ln|n| = \sin\left(\frac{\pi t}{6}\right) + C$$

$$\text{Since } n > 0 \quad n = Ae^{\sin\left(\frac{\pi t}{6}\right)}, \quad A \in \mathbb{R}$$

Solving for A using the initial conditions:

$$100\,000 = Ae^{\sin\left(\frac{\pi \times 0}{6}\right)}$$

$$A = 100\,000$$

The maximum occurs when $\sin\left(\frac{\pi t}{6}\right) = 1$ and the minimum when $\sin\left(\frac{\pi t}{6}\right) = -1$. Thus

$$n_{\max} = e^1 \times 100\,000 \approx 272\,000$$

$$n_{\min} = e^{-1} \times 100\,000 \approx 37\,000$$

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Question 13(d) (i) (2 marks)
Outcomes Assessed: *ME11-1*
Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	2
• correctly differentiates $y = xe^x$	1

Sample Answer:

$$\begin{aligned} \text{If} \quad y &= xe^x \\ \frac{dy}{dx} &= xe^x + e^x \\ &= e^x(x+1) \end{aligned}$$

When $\frac{dy}{dx} = 0$, $x = -1$ and $y = -e^{-1}$, so P is $\left(-1, -\frac{1}{e}\right)$.

Question 13(d) (ii) (1 mark)
Outcomes Assessed: *ME11-1*
Targeted Performance Bands: *E2*

Criteria	Mark
• provides correct solution	1

Sample Answer:
The largest possible domain is $\left[-\frac{1}{e}, \infty\right)$.

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Question 13(d) (iii) (2 marks)
Outcomes Assessed: *ME11-1, ME12-4*
Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	2
• calculates the gradient of the tangent to $y = f(x)$ at $x = \frac{1}{2}$	1

Sample Answer:

To find the gradient of the tangent to $y = f^{-1}(x)$ at the point $Q\left(\frac{\sqrt{e}}{2}, \frac{1}{2}\right)$, we find the reciprocal of the gradient to $y = xe^x$ when $x = \frac{1}{2}$.

When $x = \frac{1}{2}$, $\frac{dy}{dx} = \frac{3\sqrt{e}}{2}$, hence the gradient of the tangent is $\frac{2}{3\sqrt{e}}$. Thus

$$y - \frac{1}{2} = \frac{2}{3\sqrt{e}}\left(x - \frac{\sqrt{e}}{2}\right)$$
$$y - \frac{1}{2} = \frac{2x}{3\sqrt{e}} - \frac{1}{3}$$
$$y = \frac{2x}{3\sqrt{e}} + \frac{1}{6}$$

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Question 14 (15 marks)

Question 14(a) (3 marks)

Outcomes Assessed: *ME 12-4*

Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	3
• integrates correctly to obtain an equation in k	2
• correct statement equating the volumes, with the correct variables and limits	1

Sample Answer:

Equating volumes $\displaystyle\int_0^k y^2\,dx = \int_0^{\sqrt{k}} x^2\,dy$

$$\int_0^k x\,dx = \int_0^{\sqrt{k}} y^4\,dy$$

$$\left.\frac{x^2}{2}\right]_0^k = \left.\frac{y^5}{5}\right]_0^{\sqrt{k}}$$

$$\frac{k^2}{2} = \frac{k^{\frac{5}{2}}}{5}$$

$$\therefore \quad k = \frac{25}{4}$$

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Question 14(b) (i) (2 marks)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E3*

Criteria	Marks
• provides correct solution	2
• identifies the geometric series	1

Sample Answer:
We use the complementary probability. Firstly, we find $P(X \leqslant n)$.

$$\begin{aligned} P(X \leqslant n) &= P(X=1)+P(X=2)+P(X=3)+\cdots+P(X=n) \\ &= p+p(1-p)+p(1-p)^2+\cdots+p(1-p)^{n-1} \end{aligned}$$

This is a geometric series with initial term p, common ratio $(1-p)$ and n terms. Thus

$$\begin{aligned} P(X \leqslant n) &= \frac{p[1-(1-p)^n]}{1-(1-p)} \\ &= \frac{p[1-(1-p)^n]}{p} \\ &= 1-(1-p)^n \end{aligned}$$

Then

$$\begin{aligned} P(X > n) &= 1-P(X \leqslant n) \\ &= 1-[1-(1-p)^n] \\ &= (1-p)^n, \quad \text{as required} \end{aligned}$$

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Question 14(b) (ii) (2 marks)
Outcomes Assessed: *ME12-5*
Targeted Performance Bands: *E4*

Criteria	Marks
• correct solution	2
• some progress	1

Sample Answer:
From the conditional probability formula,

$$P(X > m+n \mid X > m) = \frac{P(X > m+n \cap X > m)}{P(X > m)}.$$

However, as m and n are both positive, $X > m+n$ implies $X > m$. Thus

$$P(X > m+n \mid X > m) = \frac{P(X > m+n)}{P(X > m)}.$$

From part (i),

$$\begin{aligned} P(X > m+n \mid X > m) &= \frac{(1-p)^{m+n}}{(1-p)^m} \\ &= (1-p)^n \\ &= P(X > n), \quad \text{as required} \end{aligned}$$

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Question 14(b) (iii) (1 mark)
Outcomes Assessed: *ME12-8*
Targeted Performance Bands: *E4*

Criteria	Mark
• correct reasoning referring to part(ii)	1

Sample Answer:
From part (ii),

$$P(X > m+n \mid X > m) = P(X > n).$$

In other words, even if Louis loses the first m rounds, the probability that he loses the next n rounds does not change. Thus his "luck" (probability of winning) does not increase as he continues to play the poker machine.

Question 14(c) (3 marks)
Outcomes Assessed: *ME11-2, ME11-7*
Targeted Performance Bands: *E4*

Criteria	Marks
• provides correct solution	3
• one correct value of b or equivalent merit	2
• finds $\alpha = 0$ or $\frac{2}{3}$ OR $\beta = 1$ or $-\frac{1}{3}$	1

Sample Answer:
Let $P(x) = x^3 - 2x$ and $Q(x) = (x-1)^2 + b$. Consider the function $F(x) = P(x) - Q(x)$. For the graphs to intersect exactly twice, the polynomial equation $F(x) = 0$ must have exactly one double root and one single root.

Let the roots of $F(x) = 0$ be α, α and β.

$$\begin{aligned} F(x) &= P(x) - Q(x) \\ &= x^3 - 2x - (x-1)^2 - b \\ &= x^3 - x^2 - (1+b) \end{aligned}$$

Consider $F'(x) = 3x^2 - 2x$

Since α is a double root $F'(\alpha) = 3\alpha^2 - 2\alpha = 0$

$\therefore \quad \alpha(3\alpha - 2) = 0$

$\alpha = 0$ or $\frac{2}{3}$

for $\alpha = 0$ $\quad F(0) = 0 - 0 - (1+b) = 0 \implies b = -1$

and for $\alpha = \frac{2}{3}$ $\quad F\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 - (1+b) = 0 \implies b = -\frac{31}{27}$

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Question 14(d) (4 marks)
Outcomes Assessed: *ME11-4*
Targeted Performance Bands: *E4*

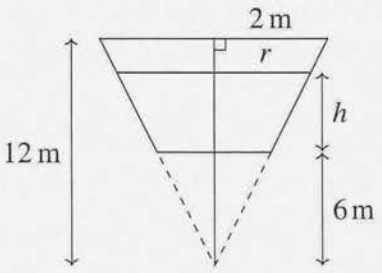
Criteria	Marks
• provides correct solution	4
• correct expression for $\frac{dh}{dV}$ or equivalent	3
• correct expression for the volume in terms of h or equivalent	2
• correct similarity relationship, diagram or expression for the chain rule	1

Sample Answer:

We need to find $\dfrac{dh}{dt} = \dfrac{dh}{dV}\dfrac{dV}{dt}$

We are given $\dfrac{dV}{dt} = -1.5$ and $V = \dfrac{\pi}{3}h\left(r^2+r+1\right)$

By similarity $\dfrac{r}{2} = \dfrac{6+h}{12} \implies r = \dfrac{6+h}{6}$



$$V = \frac{\pi}{3}h\left(r^2+r+1\right) = \frac{\pi}{3}h\left[\left(\frac{6+h}{6}\right)^2 + \frac{6+h}{6} + 1\right]$$

$$= \frac{\pi}{108}\left[108h + 18h^2 + h^3\right]$$

Differentiating wrt h: $\dfrac{dV}{dh} = \dfrac{\pi}{108}\left[108 + 36h + 3h^2\right]$

$$\therefore \quad \frac{dh}{dV} = \frac{36}{\pi(36+12h+h^2)}$$

and when $h = 1$ $= \dfrac{36}{49\pi}$

So $\dfrac{dh}{dt} = \dfrac{dh}{dV} \times \dfrac{dV}{dt} = \dfrac{36}{49\pi} \times -1.5$

$= -0.35079\ldots$ m per minute

≈ -35 cm per minute

So, when $h = 1$ m, the height is falling at a rate 35 cm per minute or
height is changing at a rate of -35 cm per minute

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