Name: _____

Teacher: _____

Merewether High School

2025

Year 12 Mathematics Advanced
TRIAL HSC

General Instructions

- Working Time – 3 hours.
- Reading time – 10 minutes.
- Write using a black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- Show relevant mathematical working, reasoning and/or calculations.
- Extra working space is provided at the end of the paper.
- Submit ALL work at the end of the examination.
- Diagrams are NOT drawn to scale.

Total Marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Answer on the multiple choice grid provided.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 7 – 33)

- Attempt Questions 11 – 33.
- Allow about 2 hours and 45 minutes for this section.

.

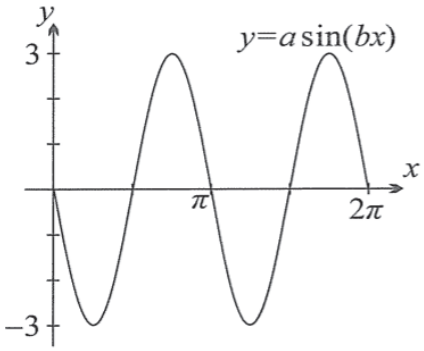
Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

Question 1

The diagram below is a graph of $y = a\sin(bx)$ in the domain $[0,2\pi]$.



Which equation describes this graph?

A. $y = 2\sin 3x$

B. $y = 3\sin 2x$

C. $y = -2\sin 3x$

D. $y = -3\sin 2x$

Question 2

Which expression is equivalent to $\cos^2\left(\frac{\pi}{2} - \theta\right) cot\ \theta$?

A. $\cos^2\theta\ cot\theta$

B. $\sin\theta\ \cos\theta$

C. $\frac{\sin^3\theta}{\cos\theta}$

D. $\sin^2\theta\ cot\theta$

Question 3

What is the gradient of the normal to the curve $f(x) = (3x-5)(x+1)$ at the y-intercept?

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. 2

Question 4

What is the value of $(\log a)^2 \div \log\left(a^2\right)$?

A. 1

B. $\frac{\log a}{2}$

C. $\log a$

D. $2\log a$

Question 5

Consider the function $f(x) = \ln(x-2) + \sqrt{3-x}$.

What is the domain of the function?

A. $(-\infty,\infty)$

B. $(0,\infty)$

C. $(3,2]$

D. $(2,3]$

Question 6

The graph $y = \dfrac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\dfrac{1}{2}$.

Which of the following gives the equation of the new function?

A. $\dfrac{y}{2} = \dfrac{3}{x-3}$

B. $2y = \dfrac{3}{x-4}$

C. $2y = \dfrac{3}{x-3}$

D. $\dfrac{y}{2} = \dfrac{3}{x-4}$

Question 7

At a particular point on a curve, $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$.

This means the curve is:

A. Decreasing and is concave down

B. Decreasing and is concave up

C. Increasing and is concave down

D. Increasing and is concave up

Question 8

The following table shows the results of tests (out of 100) in English and Science for all the students in Tilly's class.

Subject	Class mean	Class standard deviation	Tilly's mark	Tilly's z-score
English	72	8		-0.25
Science	75	6	72	

Which of the following statements is correct?

A. *Tilly had a higher mark and a higher z-score in Science than in English.*

B. *Tilly had a higher mark but a lower z-score in Science than in English.*

C. *Tilly had a lower mark and a lower z-score in Science than in English.*

D. *Tilly had a lower mark and a higher z-score in Science than in English.*

Question 9

Which of the following is equal to $\frac{4^n - 1}{2^n + 1}$?

A. $2^n + 1$

B. $2^n - 1$

C. 2^2

D. 2

Question 10

Let A and B be two independent events from a sample space.
If $P(A) = p$, $P(B) = p^2$ and $P(A) + P(B) = 1$, then $P(A' \cup B)$ is equal to

A. $1 - p - p^2$

B. $p + p^3$

C. $1 - p + p^3$

D. $1 - p - p^2 + p^3$

End of Section I

Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11 - 33

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of the Booklet.
  If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (5 marks)

a) Let $y = e\cos(3x)$. Find $\frac{dy}{dx}$. 1

……

……

……

……

……

……

b) Let $f(x) = \log_e(x^3 - 3x + 2)$. Find $f'(3)$. 2

……

……

……

……

……

……

……

c) Find $\int \frac{3x}{4 + 2x^2} dx$. 2

……

……

……

……

……

……

……

Question 12 (5 marks)

Solve the following equations, express in simplest form.

a) $|2x - 1| = 3$ 2

……

……

……

……

……

……

……

b) $x^{\ln 2} = 8$ 3

……

……

……

……

……

……

……

Question 13 (2 marks)

The function $P(x) = x^3 + bx^2 + cx + d$, has x intercepts at 4, 0 and -3. Find b, c and d.

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Question 14 (4 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X = x)$	k^2	0.16	0.18	0.3	k	0.12

a) Show that $k = 0.2$ 2

……

……

……

……

……

……

b) Calculate $E(X)$ 1

……

……

……

……

……

……

c) Calculate $Var(X)$ 1

……

……

……

……

Question 15 (6 marks)

Consider the function $f(x) = \dfrac{x}{x^2 + 1}$

a) Show that $f(x)$ is odd. 1

……

……

……

……

……

b) Find any stationary points and determine their nature. 3

……

……

……

……

……

……

……

……

……

……

……

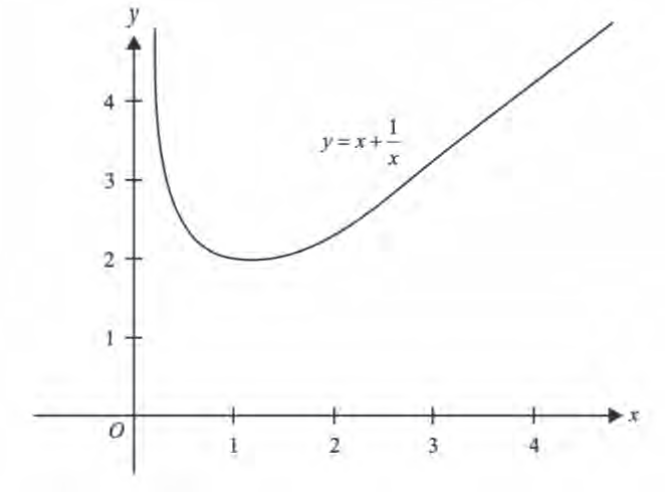
……

……

c) Sketch $y = f(x)$ showing intercepts, asymptotes and stationary points. 2

Question 16 (2 marks)

The graph of $y = x + \frac{1}{x}$ is shown over part of its domain.



Use two trapeziums of equal width to approximate the area between the curve, the x-axis and the lines $x = 1$ and $x = 3$

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Question 17 (2 marks)

The sum of the first 10 terms of an arithmetic sequence is 160.

The sum of the first 20 terms is 520.

What is the **first term** a of the sequence?

……

……

……

……

……

……

……

……

……

……

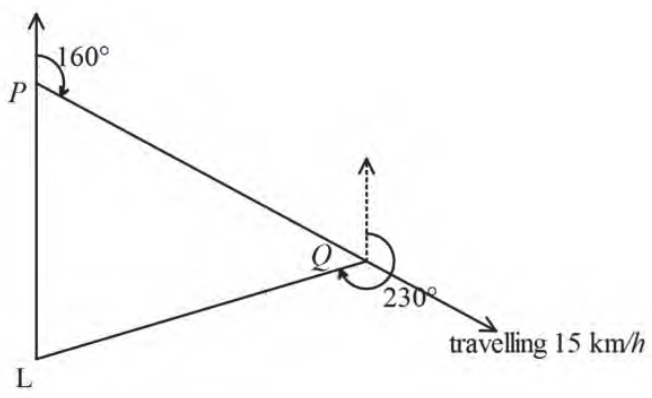
……

……

……

Question 18 (6 marks)

A ship is sailing at 15 km/h on a bearing of 160°T. At 9:00 am the ship is at P, and the lighthouse, L is due south of the ship. At 9:40 am the ship is at Q, and the lighthouse is on a bearing of 230°T.



a) Show that $\angle PQL = 110°$ 1

……

……

……

b) Show that the distance PL is 12.3 km (correct to 1 decimal place). 2

……

……

……

……

c) Find the time, correct to the nearest minute, at which the lighthouse will be due west of the ship. 3

……

……

……

……

……

……

Question 19 (4 marks)

A survey of all the Zoology students at a local university showed that $\frac{3}{4}$ of them had a pet cat, $\frac{3}{8}$ of them had a pet dog and $\frac{1}{4}$ had both a pet cat and a pet dog. 25 students had either a pet cat or a pet dog but not both types of pet.

a) Show that a total of 40 Zoology students were surveyed. 2

……

……

……

……

b) If two of the zoology students are chosen at random, find in simplest fraction form the probability that neither of these students has a pet dog. 2

……

……

……

……

Question 20 (2 marks)

Show that $\log_e(sec\ \theta + \tan\theta) + \log_e(sec\ \theta - \tan\theta) = \log_e(1)$ for $0 < \theta < \frac{\pi}{2}$.

……

……

……

……

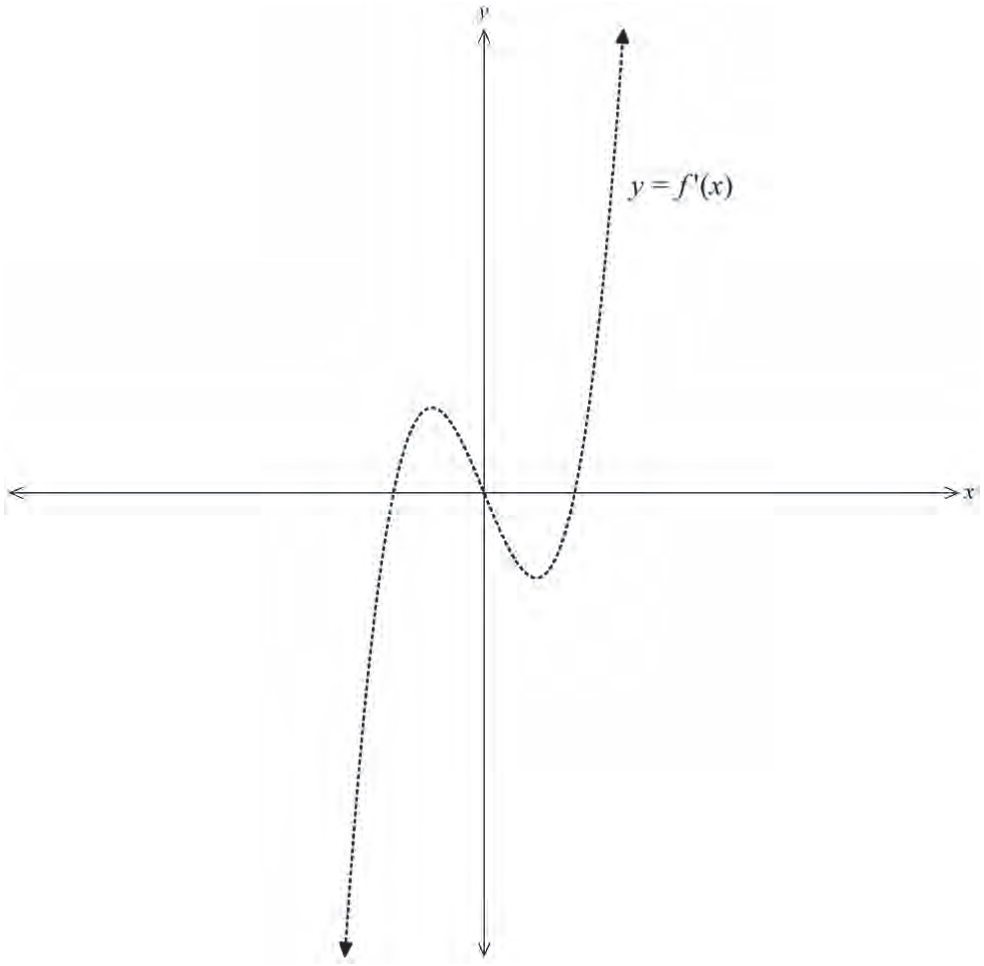
……

……

……

Question 21 (2 marks)

The following shows the graph of $f'(x)$.



On the above axis, draw a possible graph of $f(x)$

Question 22 (2 marks)

A company offers a special deal for a machine lease: a business can lease the machine for an initial payment of $1200. Each year after that, the payment is reduced by 15% of the previous year's payment. The lease continues indefinitely.

a) Find the cost of the lease in the 6th year. 1

……

……

……

……

……

……

……

……

b) Find the total amount the company will receive from one lease. 1

……

……

……

……

……

……

……

……

……

……

Question 23 (3 marks)

a) Evaluate $\int_0^{\frac{\pi}{3}} \sin(x)\,dx$. 1

……

……

……

……

b) Hence, or otherwise, find all values of k such that $\int_0^{\frac{\pi}{3}} \sin(x)\,dx = \int_k^{\frac{\pi}{2}} \cos(x)\,dx$ 2

where $0 < k < 2\pi$.

……

……

……

……

……

……

……

……

……

……

……

……

Question 24 (3 marks)

Find in gradient intercept form the equation of the tangent to the curve $y = \sqrt{6x+4} + e^{\frac{1}{2}x}$ at the point on the curve where $x = 0$.

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Question 25 (3 marks)

8 students in an Advanced Mathematics class sat term tests in *Functions* and *Calculus*.

The table below shows their marks out of 100 in each test.

Student	A	B	C	D	E	F	G	H
Functions (x)	63	59	69	83	77	75	85	
Calculus (y)	58	62	70	72	76	76	78	84

Unfortunately, the *Functions* mark for student H has been lost. However, it is known that the correlation coefficient between *Functions* mark and *Calculus* marks is $r = 0.9$ and the equation of the least squares regression line of *Calculus* marks on *Functions* marks is $y = 0.72x + 18$

a) Use the mean $\bar{y} = 72$ of the *Calculus* marks and the least squares regression line to find the mean $\bar{x}$ of the *Functions* marks, and hence the missing *Functions* mark for student H. 2

……

……

……

……

……

……

……

……

b) Another student in the same class sat the *Functions* test and achieved a mark of 93 but missed the *Calculus* test. Explain whether or not you would be justified statistically in using their *Functions* mark and the given least squares regression line to estimate their *Calculus* mark. 1

……

……

……

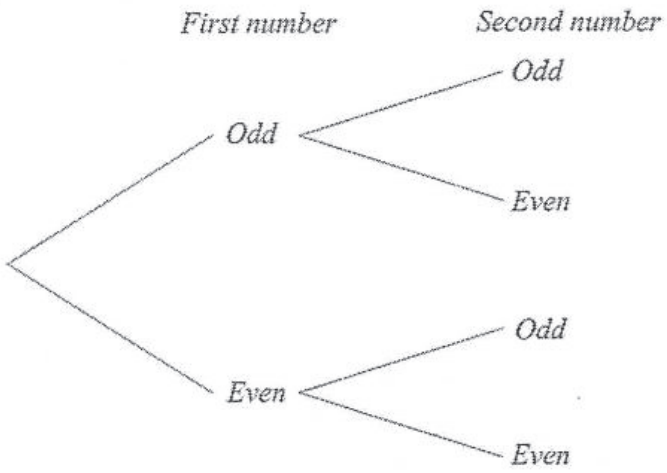
……

……

Question 26 (5 marks)

Two numbers are chosen without replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9.

a) Complete the probability tree diagram below. 1



b) Find, in simplest form, the probability that the product of the two numbers chosen is even. 2

……

……

……

……

……

……

c) Given that the product of the two numbers chosen is even, find the probability that their sum is even, expressing your answer is simplest fraction form. 2

……

……

……

……

……

Question 27 (4 marks)

Zenthralium is a rare radioactive substance that decays with a highly toxic residue.

The rate of change is given by $\frac{dM}{dt} = -kM$,

where k is a positive constant and M is the mass present.

a) Show that $M = M_0 e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$. 1

……

……

……

……

……

b) The half-life of Zenthralium is 29 years. This means it takes 29 years for 100g to decay to 50g. Find the value of k correct to 3 significant figures. 2

……

……

……

……

……

c) A decaying bag of Zenthralium is found illegally dumped at a landfill site. It is weighed and its mass is 12 kg. Calculate the original mass if it was dumped 10 years ago. Give your answer correct to 2 decimal places. 1

……

……

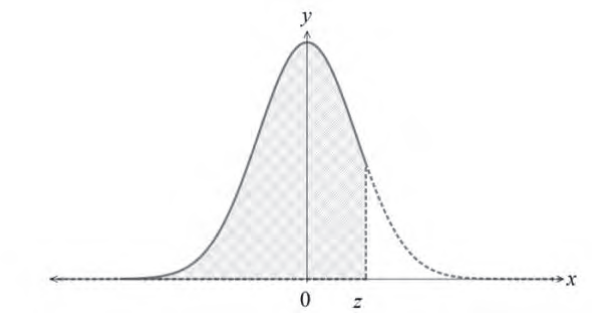
……

……

Question 28 (2 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table to the right gives the probability that this random variable is less than z.

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



A study with 500 participants found that the circumference of their heads was normally distributed with mean 56.0 cm and standard deviation 1.5 cm.

By calculating z-scores, find the number of participants whose head circumference was between 57.2 cm and 58.4 cm.

z-score	Probability
0	0.5
0.2	0.57926
0.4	0.65542
0.6	0.72575
0.8	0.78814
1	0.84134
1.2	0.88493
1.4	0.91924
1.6	0.94520
1.8	0.96407
2	0.97725
2.2	0.98610
2.4	0.99180

……

……

……

……

……

……

……

……

……

……

……

Question 29 (5 marks)

Tristan has a tin of paint in the shape of a cylinder. The cylinder has a radius of r cm and a height of h cm. Tristan knows the sum of the radius and height is 24 cm.

a) Show the volume of the paint tin is found by: 2

$$V = 24\pi r^2 - \pi r^3$$

……

……

……

……

……

……

b) Find the maximum capacity of the tin of paint in litres. 3

……

……

……

……

……

……

……

……

……

……

……

……

Question 30 (6 marks)

Consider the probability density function below that models the lifespan of a certain brand of washing machine, in years.

$$f(w) = \begin{cases} \dfrac{k}{\sqrt{9+2w}} & 0 \leq w \leq 8 \\ 0, & \text{otherwise} \end{cases}$$

a) Find the value of k. 2

……

……

……

……

……

……

b) Find the equation of the cumulative distribution function. 2

……

……

……

……

……

……

c) Hence, or otherwise, find the median lifespan of this brand of washing machine. 2

……

……

……

……

……

……

……

Question 31 (5 marks)

The depth of water (in metres) in a lagoon in the coastal town of Schoolies is given by:

$$D = 8 + 2\cos\left(\frac{4\pi t}{25}\right) \text{ for } 0 \leq t \leq 25.$$

where t is the time in hours after 12 noon .

a) What is the depth of water at 12 noon? 1

……

……

……

……

……

……

b) What is the first time after 12 midnight that the water level will be at the depth of 7m? 2

……

……

……

……

……

……

……

……

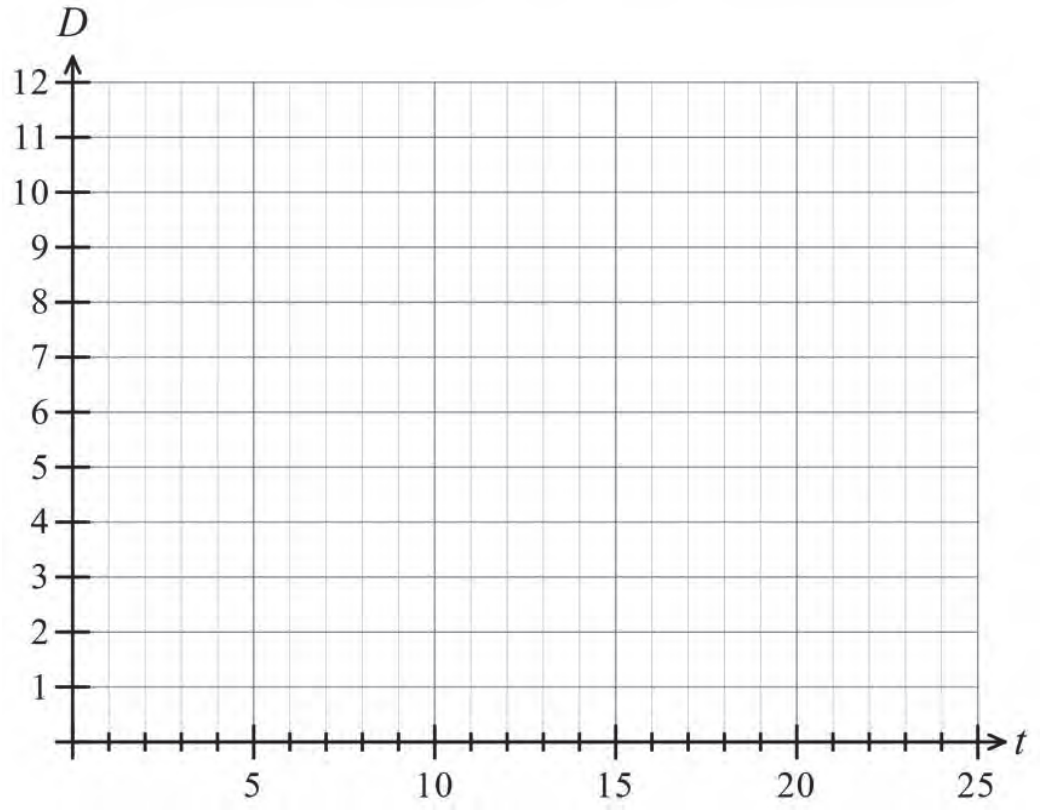
……

……

Question 31 continues on the next page.

c) Sketch the curve on the axes below, showing the maximum and minimum depths of the lagoon during the time period $0 \leq t \leq 25$. 2

Draw your sketch below:



Question 32 (5 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres.

The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$.

Initially its velocity is -6 m/s and its displacement is 5 m.

a) Find the equation for the displacement (x) of the particle in terms of t. 3

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

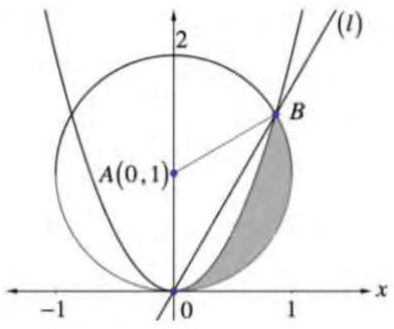
……

Question 32 continues on the next page

b) At what time and where does the particle come to rest? 2

Question 33 (7 marks)

A circle centred at A with radius 1 unit intersects the parabola $y = 2x^2$ at the origin O and at the point B. The line l passes through O and B as shown in the diagram.



a) Show the coordinates of B are $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$. 2

……

……

……

……

……

……

……

……

b) Find the angle OB makes with the positive x axis. 1

……

……

……

……

Question 33 continues on the next page

c) Show that $\angle OAB = \frac{2\pi}{3}$. 1

……

……

……

……

……

……

d) Find the shaded area bounded by the circle and the parabola in the first quadrant as shown in the diagram. 3

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

End of paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Name: Solutions

Teacher: _____

Merewether High School

2025

Year 12 Mathematics Advanced
TRIAL HSC

General Instructions

- Working Time – 3 hours.
- Reading time – 10 minutes.
- Write using a black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- Show relevant mathematical working, reasoning and/or calculations.
- Extra working space is provided at the end of the paper.
- Submit ALL work at the end of the examination.
- Diagrams are NOT drawn to scale.

Total Marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Answer on the multiple choice grid provided.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 7 – 33)

- Attempt Questions 11 – 33.
- Allow about 2 hours and 45 minutes for this section.

.

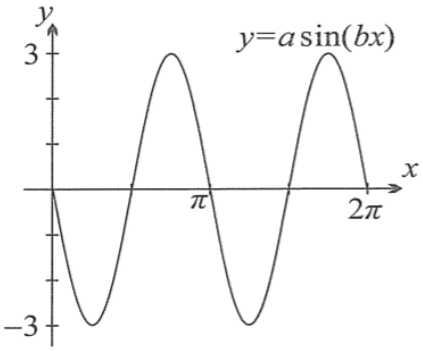
Section I

10 Marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10

Question 1

The diagram below is a graph of $y = a\sin(bx)$ in the domain $[0,2\pi]$.



Which equation describes this graph?

A. $y = 2\sin 3x$

B. $y = 3\sin 2x$

C. $y = -2\sin 3x$

D. $y = -3\sin 2x$

$a = -3$

$\frac{2\pi}{b} = \pi$

$b = 2$

Question 2

Which expression is equivalent to $\cos^2\left(\frac{\pi}{2} - \theta\right)\cot\theta$?

A. $\cos^2\theta\ \cot\theta$

B. $\sin\theta\ \cos\theta$

C. $\frac{\sin^3\theta}{\cos\theta}$

D. $\sin^2\theta\ \cot\theta$

$\sin^2\theta \times \frac{\cos\theta}{\sin\theta} = \sin\theta\cos\theta$

Question 3

What is the gradient of the normal to the curve $f(x) = (3x-5)(x+1)$ at the y-intercept?

$= 3x^2 - 2x - 5$

$f'(x) = 6x - 2$

$f'(0) = -2$

normal $= \frac{1}{2}$

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -2

D. 2

Question 4

What is the value of $(\log a)^2 \div \log(a^2)$?

$= \frac{(\log a)^2}{2 \log a}$

$= \frac{\log a}{2}$

A. 1

B. $\frac{\log a}{2}$

C. $\log a$

D. $2\log a$

Question 5

Consider the function $f(x) = \ln(x-2) + \sqrt{3-x}$.

What is the domain of the function?

$x - 2 > 0$

$x > 2$

$3 - x \geq 0$

$-x \geq -3$

$x \leq 3$

A. $(-\infty, \infty)$

B. $(0, \infty)$

C. $(3,2]$

D. $(2,3]$

Question 6

The graph $y = \dfrac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\dfrac{1}{2}$.

Which of the following gives the equation of the new function?

A. $\dfrac{y}{2} = \dfrac{3}{x-3}$

B. $2y = \dfrac{3}{x-4}$

C. $2y = \dfrac{3}{x-3}$

D. $\dfrac{y}{2} = \dfrac{3}{x-4}$

$2y = \dfrac{3}{(x-4)+1}$

$2y = \dfrac{3}{x-3}$

Question 7

At a particular point on a curve, $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$.

This means the curve is:

A. Decreasing and is concave down

B. Decreasing and is concave up

C. Increasing and is concave down

D. Increasing and is concave up

Question 8

The following table shows the results of tests (out of 100) in English and Science for all the students in Tilly's class.

Subject	Class mean	Class standard deviation	Tilly's mark	Tilly's z-score
English	72	8	$72 - 2 = 70$	-0.25
Science	75	6	72	$\frac{72-75}{6} = -\frac{1}{2}$

Which of the following statements is correct?

A. *Tilly had a higher mark and a higher z-score in Science than in English.*

B. *Tilly had a higher mark but a lower z-score in Science than in English.*

C. *Tilly had a lower mark and a lower z-score in Science than in English.*

D. *Tilly had a lower mark and a higher z-score in Science than in English.*

Question 9

Which of the following is equal to $\frac{4^n - 1}{2^n + 1}$?

A. $2^n + 1$

B. $2^n - 1$

C. 2^2

D. 2

$$\frac{(2^2)^n - 1}{2^n + 1} = \frac{(2^n)^2 - 1}{2^n + 1}$$

$$= \frac{(2^n - 1)(2^n + 1)}{(2^n + 1)}$$

$$= 2^n - 1$$

Question 10

Let A and B be two independent events from a sample space.

If $P(A) = p$, $P(B) = p^2$ and $P(A) + P(B) = 1$, then $P(A' \cup B)$ is equal to

A. $1 - p - p^2$

B. $p + p^3$

C. $1 - p + p^3$

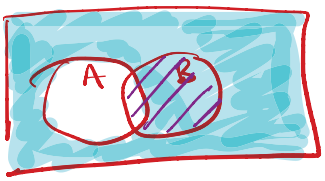
D. $1 - p - p^2 + p^3$

$P(A \cap B) = P(A) \times P(B)$
$= p \times p^2$
$= p^3$
since events are independent

End of Section I

$P(A' \cap B) = P(A') \times P(B)$
$= (1-p) \times p^2$
$= p^2 - p^3$

$P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
$= (1-p) + p^2 - (p^2 - p^3)$
$= 1 - p + p^2 - p^2 + p^3$
$= 1 - p + p^3$



Mathematics Advanced

Section II Answer Booklet

90 marks

Attempt Questions 11 - 33

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of the Booklet.
  If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (5 marks)

a) Let $y = e\cos(3x)$. Find $\frac{dy}{dx}$. 1

$\frac{dy}{dx} = -3e\sin(3x)$

b) Let $f(x) = \log_e(x^3 - 3x + 2)$. Find $f'(3)$. 2

$f'(x) = \frac{3x^2 - 3}{x^3 - 3x + 2}$ (1)

$f'(3) = \frac{3 \times 3^2 - 3}{3^3 - 3 \times 3 + 2}$

$= \frac{24}{20}$ (1)

$= \frac{6}{5}$

c) Find $\int \frac{3x}{4 + 2x^2} dx$. 2

$= \frac{3}{4}\int \frac{4x}{4 + 2x^2} dx$ (1) recognise log

$= \frac{3}{4}\ln(4 + 2x^2) + C$ (1)

Question 12 (5 marks)

Solve the following equations, express in simplest form.

a) $|2x - 1| = 3$ 2

$x > 0 \quad 2x - 1 = 3$

$2x = 4$

$x = 2$ (1)

$x < 0 \quad -(2x - 1) = 3$

$-2x + 1 = 3$

$-2x = 2$

$x = -1$ (1)

b) $x^{\ln 2} = 8$ 3

$\ln(x^{\ln 2}) = \ln 2^3$ (1) log

$\ln 2(\ln x) = 3\ln 2$

$\ln x = 3$ (1)

$e^3 = x$ (1)

Question 13 (2 marks)

The function $P(x) = x^3 + bx^2 + cx + d$, has x intercepts at 4, 0 and -3. Find b, c and d.

$f(x) = x(x-4)(x+3)$ (1)

$= x(x^2 - x - 12)$

$= x^3 - x^2 - 12x$ (1)

$\therefore b = -1$

$c = -12$

$d = 0$

Question 14 (4 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X = x)$	k^2	0.16	0.18	0.3	k	0.12

a) Show that $k = 0.2$ 2

$k^2 + k = 1 - (0.16 + 0.18 + 0.3 + 0.12)$

$k^2 + k = 0.24$

$k^2 + k - 0.24 = 0$ (1)

$k = \frac{-1 \pm \sqrt{1^2 + 4 \times 1 \times 0.24}}{2 \times 1}$

$= 0.2, -1.2$ $k > 0$ (1)

$\therefore k = 0.2$

b) Calculate $E(X)$ 1

$(0 \times 0.04) + 2 \times 0.16 + 4 \times 0.18 + 5 \times 0.3 + 8 \times 0.2 + 9 \times 0.12$

$= 5.22$

c) Calculate $Var(X)$ 1

$= \sum x^2 p(x) - \mu^2$ $\mu = E(X)$

$= (0^2 \times 0.04) + (2^2 \times 0.16) + (4^2 \times 0.18)$

$+ (5^2 \times 0.3) + (8^2 \times 0.2) + (9^2 \times 0.12) - (5.22)^2$

$= 6.29$

Question 15 (6 marks)

Consider the function $f(x) = \dfrac{x}{x^2+1}$

a) Show that $f(x)$ is odd. 1

$f(-x) = \dfrac{-x}{(-x)^2+1}$ $\quad -f(x) = -\dfrac{x}{x^2+1}$

$= \dfrac{-x}{x^2+1}$

$= -f(x)$ since $f(-x) = -f(x)$, $f(x)$ is odd

b) Find any stationary points and determine their nature. 3

stat pts when $f'(x) = 0$

$y' = \dfrac{(x^2+1) - 2x(x)}{(x^2+1)^2}$

$= \dfrac{x^2+1-2x^2}{(x^2+1)^2}$

$= \dfrac{-x^2+1}{(x^2+1)^2}$ (1)

$-x^2+1 = 0$

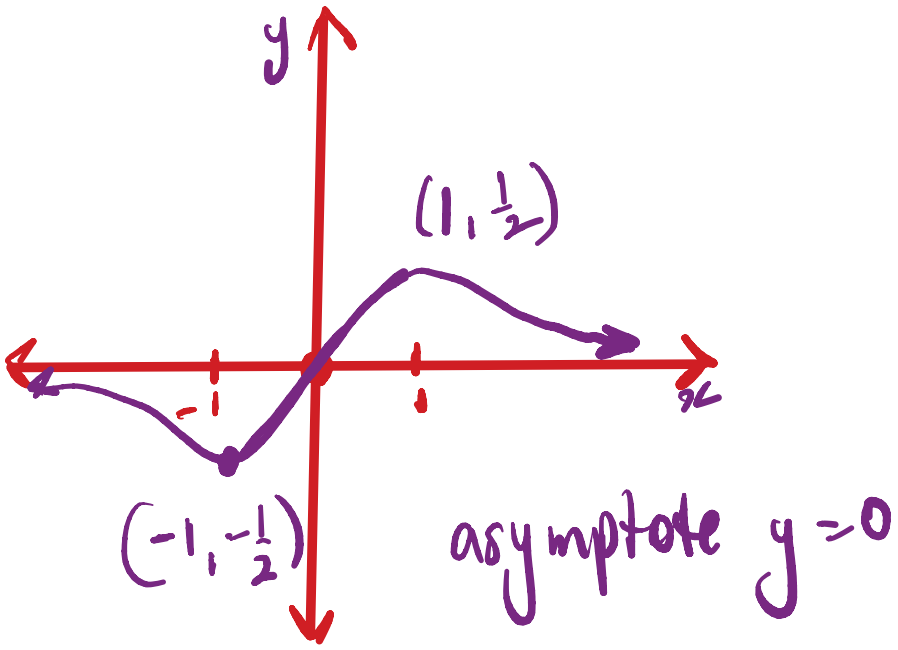
$x = \pm 1$, $y = \pm\frac{1}{2}$ (1)

test nature

x	-2	-1	0	1	2
y'	-	0	+	0	-

min max (1)

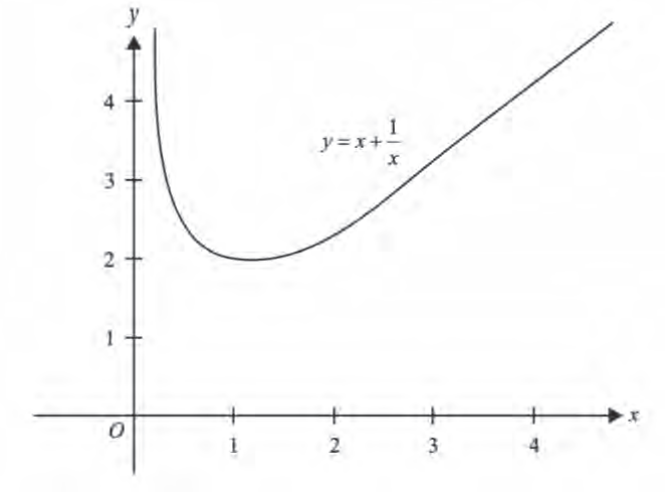
c) Sketch $y = f(x)$ showing intercepts, asymptotes and stationary points. 2



shape (1)
points (1)

Question 16 (2 marks)

The graph of $y = x + \frac{1}{x}$ is shown over part of its domain.



Use two trapeziums of equal width to approximate the area between the curve, the x-axis and the lines $x = 1$ and $x = 3$

x	1	2	3
y	2	$2\frac{1}{2}$	$3\frac{1}{3}$

(1)

$A = \frac{h}{2} + (1st + 2 \times middles + last)$

$= \frac{1}{2}\left(2 + 2 \times \frac{5}{2} + \frac{10}{3}\right)$

$= \frac{1}{2}\left(2 + 5 + \frac{10}{3}\right)$ (1)

$= \frac{1}{2}\left(\frac{31}{3}\right)$

$= \frac{31}{6}$

$= 5\frac{1}{6}$

Question 17 (2 marks)

The sum of the first 10 terms of an arithmetic sequence is 160.

The sum of the first 20 terms is 520.

What is the **first term** a of the sequence?

$S_n = \frac{n}{2}(2a + (n-1)d)$

S_{10}: $160 = 5(2a + 9d)$

$32 = 2a + 9d$ eqn ①

S_{20}: $520 = 10(2a + 19d)$ ① eqns

$52 = 2a + 19d$ eqn ②

② − ① $20 = 10d$

$d = 2$

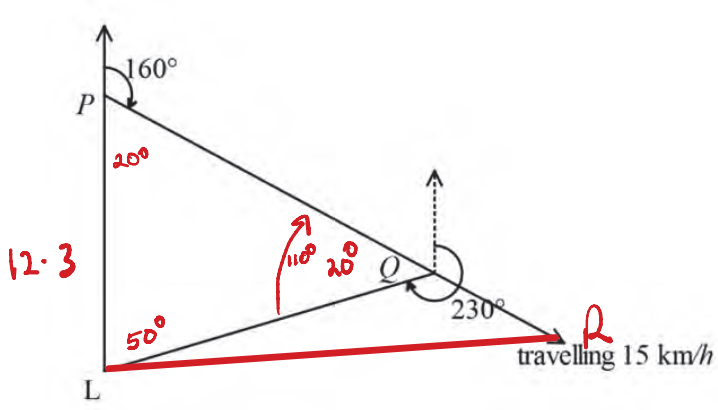
Subst in ① $32 = 2a + 18$

$2a = 14$

$a = 7$ ① soln's

Question 18 (6 marks)

A ship is sailing at 15 km/h on a bearing of 160°T. At 9:00 am the ship is at P, and the lighthouse, L is due south of the ship. At 9:40 am the ship is at Q, and the lighthouse is on a bearing of 230°T.



a) Show that $\angle PQL = 110°$ 1

$\angle PQL = 360° - (230° + 20°)$

$= 360° - 250°$

$= 110°$

b) Show that the distance PL is 12.3 km (correct to 1 decimal place). 2

$PQ = 15 \times \frac{40}{60} = 10\text{ km}$ (1)

$\frac{PL}{\sin 110°} = \frac{10}{\sin 50°}$

$PL = \frac{10 \sin 110°}{\sin 50°} \approx 12.27\text{km} \doteq 12.3$ (1)

c) Find the time, correct to the nearest minute, at which the lighthouse will be due west of the ship. 3

$\cos 20° = \frac{12.3}{PR}$ Right $\angle \Delta$

$PR = \frac{12.3}{\cos 20°} = 13\text{ km}$ (1)

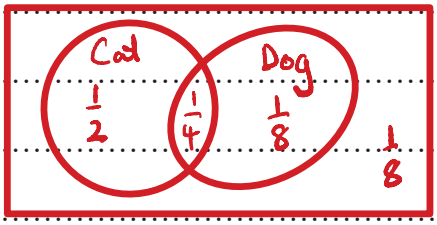
$t = \frac{D}{S} = \frac{13}{15} = 52\text{ mins}$ (1)

$\therefore$ 9:52 (1)

Question 19 (4 marks)

A survey of all the Zoology students at a local university showed that $\frac{3}{4}$ of them had a pet cat, $\frac{3}{8}$ of them had a pet dog and $\frac{1}{4}$ had both a pet cat and a pet dog. 25 students had either a pet cat or a pet dog but not both types of pet.

a) Show that a total of 40 Zoology students were surveyed. 2



(1)

$25 = \left(\frac{1}{2} + \frac{1}{8}\right)n$

$25 = \frac{5n}{8}$ (1)

$n = \frac{25 \times 8}{5}$

$= 40$

b) If two of the zoology students are chosen at random, find in simplest fraction form the probability that neither of these students has a pet dog. 2

$\frac{5}{8} \times 40 = 25$ students do not have a dog (1)

P(neither dog) $= \frac{25}{40} \times \frac{24}{39} = \frac{5}{13}$ (1)

Question 20 (2 marks)

Show that $\log_e(sec\ \theta + \tan\theta) + \log_e(sec\ \theta - \tan\theta) = \log_e(1)$ for $0 < \theta < \frac{\pi}{2}$.

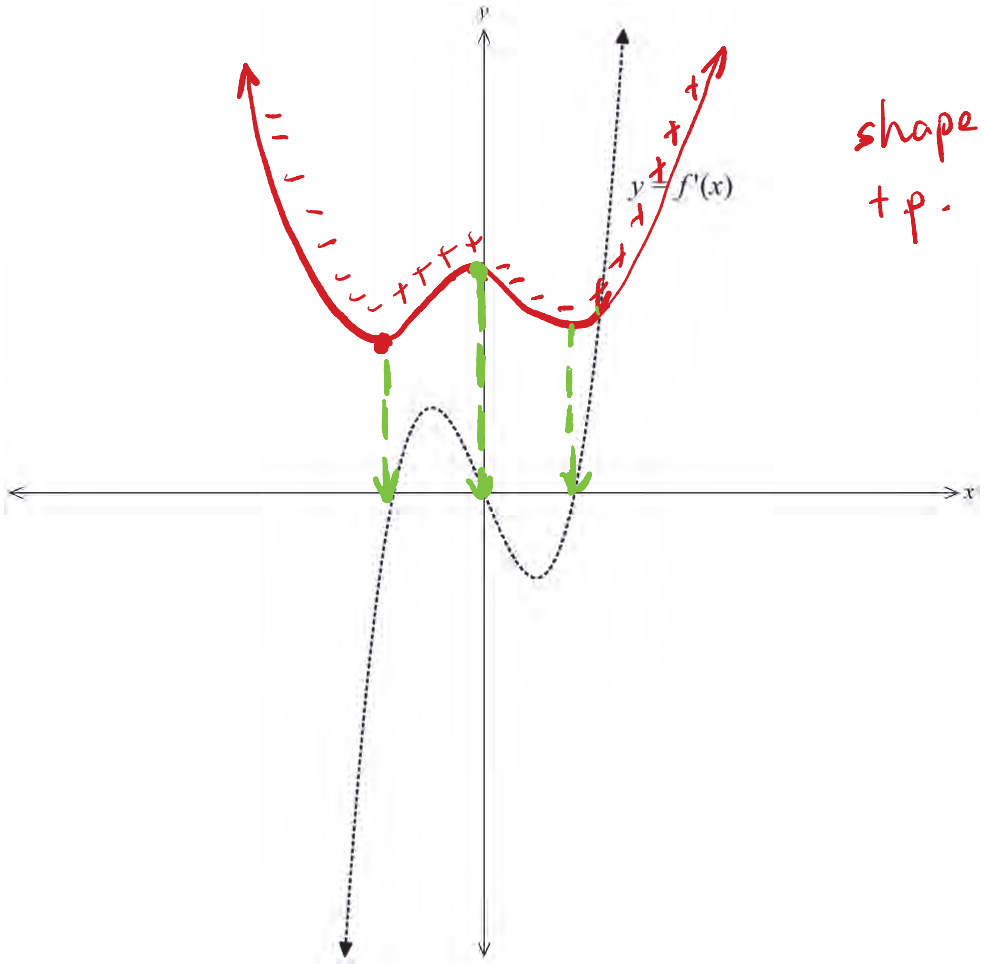
$\log_e\left((\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\right)$ (1)

$= \log_e(\sec^2\theta - \tan^2\theta)$ (1)

$= \log_e 1$

Question 21 (2 marks)

The following shows the graph of $f'(x)$.



shape (1)

+p. (1)

On the above axis, draw a possible graph of $f(x)$

Question 22 (2 marks)

A company offers a special deal for a machine lease: a business can lease the machine for an initial payment of $1200. Each year after that, the payment is reduced by 15% of the previous year's payment. The lease continues indefinitely.

a) Find the cost of the lease in the 6th year. 1

$S_n = 1200 + 1200 \times 0.85 + 1200 \times 0.85^2 + \dots$

$a = 1200 \qquad r = 0.85$

$T_n = ar^{n-1}$

$T_6 = 1200 \times 0.85^5$ (1)

$= \$532.45$

b) Find the total amount the company will receive from one lease. 1

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{1200}{1-0.85}$ (1)

$= \$8000$

Question 23 (3 marks)

a) Evaluate $\int_0^{\frac{\pi}{3}} \sin(x)\,dx$. 1

$= \left[-\cos x\right]_0^{\frac{\pi}{3}}$

$= -\cos\frac{\pi}{3} + \cos 0$

$= -\frac{1}{2} + 1$

$= \frac{1}{2}$

b) Hence, or otherwise, find all values of k such that $\int_0^{\frac{\pi}{3}} \sin(x)\,dx = \int_k^{\frac{\pi}{2}} \cos(x)\,dx$ 2

where $0 < k < 2\pi$.

$\frac{1}{2} = \left[\sin(x)\right]_k^{\frac{\pi}{2}}$

$\sin\left(\frac{\pi}{2}\right) - \sin(k) = \frac{1}{2}$ (1)

$1 - \sin(k) = \frac{1}{2}$

$\sin(k) = \frac{1}{2}$

$\therefore k = \frac{\pi}{6}, \frac{5\pi}{6}$ (1)

Question 24 (3 marks)

Find in gradient intercept form the equation of the tangent to the curve $y = \sqrt{6x+4} + e^{\frac{1}{2}x}$ at the point on the curve where $x = 0$.

$y = (6x+4)^{1/2} + e^{\frac{1}{2}x}$

$y' = 6 \times \frac{1}{2}(6x+4)^{-1/2} + \frac{1}{2}e^{\frac{1}{2}x}$

$= \frac{3}{\sqrt{6x+4}} + \frac{e^{\frac{1}{2}x}}{2}$ (1)

$y'(0) = \frac{3}{\sqrt{4}} + \frac{1}{2}$

$= 2$ (1)

at $x = 0$ $y = 2 + 1 = 3$

$y - 3 = 2(x - 0)$

$y - 3 = 2x$ (1)

$y = 2x + 3$

Question 25 (3 marks)

8 students in an Advanced Mathematics class sat term tests in *Functions* and *Calculus*.

The table below shows their marks out of 100 in each test.

Student	A	B	C	D	E	F	G	H
Functions (x)	63	59	69	83	77	75	85	
Calculus (y)	58	62	70	72	76	76	78	84

Unfortunately, the *Functions* mark for student H has been lost. However, it is known that the correlation coefficient between *Functions* mark and *Calculus* marks is $r = 0.9$ and the equation of the least squares regression line of *Calculus* marks on *Functions* marks is $y = 0.72x + 18$

a) Use the mean $\bar{y} = 72$ of the *Calculus* marks and the least squares regression line to find the mean $\bar{x}$ of the *Functions* marks, and hence the missing *Functions* mark for student H. 2

$72 = 0.72 \times \bar{x} + 18$

$\bar{x} = \frac{72 - 18}{0.72}$

$= 75$ (1)

$\text{Mark}_H = 8 \times 75 - (63 + 59 + 69 + 83 + 77 + 75 + 85)$

$= 89$ (1)

b) Another student in the same class sat the *Functions* test and achieved a mark of 93 but missed the *Calculus* test. Explain whether or not you would be justified statistically in using their *Functions* mark and the given least squares regression line to estimate their *Calculus* mark. 1

No extrapolated data

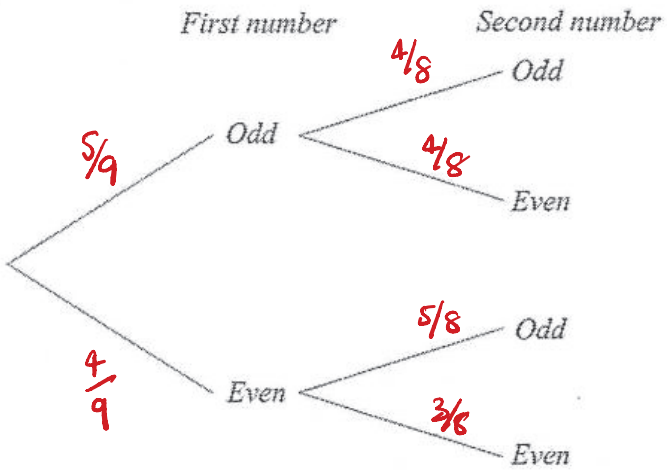
Although $r = 0.9 \Rightarrow$ strong positive linear relationship.

but outside given data

Question 26 (5 marks)

Two numbers are chosen without replacement from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9.

a) Complete the probability tree diagram below. 1



b) Find, in simplest form, the probability that the product of the two numbers chosen is even. 2

P(product even) = 1 − P(odd × odd)

$= 1 - \frac{5}{9} \times \frac{4}{8}$

$= 1 - \frac{20}{72}$

$= \frac{52}{72}$

$= \frac{13}{18}$

c) Given that the product of the two numbers chosen is even, find the probability that their sum is even, expressing your answer is simplest fraction form. 2

P(sum even ∩ product even)

= P(sum is even | product is even) × P(product even)

$\frac{4}{9_3} \times \frac{3}{8_2}$ = P(sum even | product even) = $\frac{13}{18}$

$P(\text{sum even} \mid \text{product even}) = \frac{\frac{1}{6}}{\frac{13}{18}} = \frac{3}{13}$

Question 27 (4 marks)

Zenthralium is a rare radioactive substance that decays with a highly toxic residue.

The rate of change is given by $\frac{dM}{dt} = -kM$,

where k is a positive constant and M is the mass present.

a) Show that $M = M_0 e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$. 1

$\frac{dM}{dt} = M_0 \cdot -k e^{-kt}$

$= -k(M_0 e^{-kt})$

$= -kM$

b) The half-life of Zenthralium is 29 years. This means it takes 29 years for 100g to decay to 50g. Find the value of k correct to 3 significant figures. 2

$M = M_0 e^{-kt}$

$\frac{1}{2} = e^{-k \times 29}$ (1) eqn

$\log_e \frac{1}{2} = -29k$ (1) solve

$k = 0.0239$

c) A decaying bag of Zenthralium is found illegally dumped at a landfill site. It is weighed and its mass is 12 kg. Calculate the original mass if it was dumped 10 years ago. Give your answer correct to 2 decimal places. 1

-0.0239×10

$12 = M_0 e$

$M_0 = \frac{12}{e^{-0.239}}$

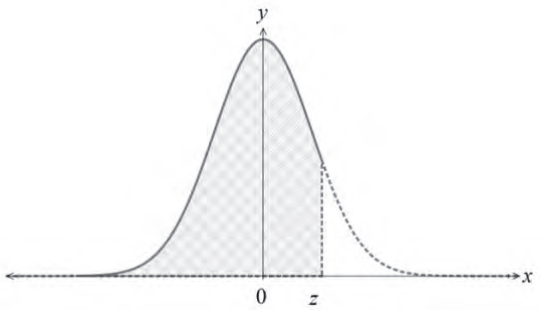
$= 15.2397\ldots$

$= 15.24$ kg

Question 28 (2 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table to the right gives the probability that this random variable is less than z.

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



z-score	Probability
0	0.5
0.2	0.57926
0.4	0.65542
0.6	0.72575
0.8	0.78814
1	0.84134
1.2	0.88493
1.4	0.91924
1.6	0.94520
1.8	0.96407
2	0.97725
2.2	0.98610
2.4	0.99180

A study with 500 participants found that the circumference of their heads was normally distributed with mean 56.0 cm and standard deviation 1.5 cm.

By calculating z-scores, find the number of participants whose head circumference was between 57.2 cm and 58.4 cm.

$z = \frac{x-\mu}{\sigma}$

$= \frac{58.4-56}{1.5}$ $\quad = \frac{57.2-56}{1.5}$ ① both z-scores

$= 1.6$ $\quad = 0.8$

$P(58.4) = 0.94520$ $\quad P(57.2) = 0.78814$

$P(57.2 < x < 58.4) = 0.94520 - 0.78814$

$= 0.15706$

$\doteq 15.7\%$ ① probability

$\therefore$ participants $= 15.7\% \times 500$

$= 78.53$

$\doteq 78$ people

Question 29 (5 marks)

Tristan has a tin of paint in the shape of a cylinder. The cylinder has a radius of r cm and a height of h cm. Tristan knows the sum of the radius and height is 24 cm.

a) Show the volume of the paint tin is found by: 2

$$V = 24\pi r^2 - \pi r^3$$

$r + h = 24$

$h = 24 - r$ (1)

$V = \pi r^2 h$

$= \pi r^2 (24 - r)$

$= 24\pi r^2 - \pi r^3$ (1)

b) Find the maximum capacity of the tin of paint in litres. 3

$V' = 48\pi r - 3\pi r^2$

stat pt ad $V' = 0$

$48\pi r - 3\pi r^2 = 0$

$3\pi r^2 = 48\pi r$

$r = 16$ (1)

$V'' = 48\pi - 6\pi r$

$V''(16) = 48\pi - 6\pi \times 16 < 0$ ∴ concave down max pt (1)

$V = \pi \times 16^2 \times 8 = 2048\pi \text{ cm}^3$

$= 2048\pi \text{ mL}$

$= 6.433981755 \text{ L}$

$\doteq 6.4 \text{ L}$ (1)

Question 30 (6 marks)

Consider the probability density function below that models the lifespan of a certain brand of washing machine, in years.

$$f(w) = \begin{cases} \dfrac{k}{\sqrt{9+2w}} & 0 \le w \le 8 \\ 0, & \text{otherwise} \end{cases}$$

a) Find the value of k. 2

$\int_0^8 \frac{k}{\sqrt{9+2w}}\,dw = 1$

$\int_0^8 k(9+2w)^{-1/2}\,dw = 1$ (1)

$k\left[\frac{(9+2w)^{1/2}}{\frac{1}{2}\times 2}\right]_0^8 = 1$

$k\left[(9+2w)^{1/2}\right]_0^8 = 1$

$k(5-3) = 1$ (1)

$2k = 1 \quad k = \frac{1}{2}$

b) Find the equation of the cumulative distribution function. 2

$F(t) = \int_0^w \frac{\frac{1}{2}}{\sqrt{9+2t}}\,dt = \frac{1}{2}\int_0^w (9+2t)^{-1/2}$

$= \frac{1}{2}\left[(9+2t)^{1/2}\right]_0^w$ (1)

$F(w) = \frac{1}{2}\left(\sqrt{9+2w} - \sqrt{9}\right)$

$= \frac{\sqrt{9+2w}}{2} - \frac{3}{2}$ (1)

c) Hence, or otherwise, find the median lifespan of this brand of washing machine. 2

$F(w) = \frac{1}{2}$

$\frac{1}{2}\sqrt{9+2w} - \frac{3}{2} = \frac{1}{2}$ (1)

$\sqrt{9+2w} - 3 = 1$

$\sqrt{9+2w} = 4$

$9 + 2w = 16$

$w = \frac{7}{2}$ (1)

∴ median lifespan $3\frac{1}{2}$ yrs

Question 31 (5 marks)

The depth of water (in metres) in a lagoon in the coastal town of Schoolies is given by:

$$D = 8 + 2\cos\left(\frac{4\pi t}{25}\right) \text{ for } 0 \leq t \leq 25.$$

where t is the time in hours after 12 noon .

a) What is the depth of water at 12 noon? 1

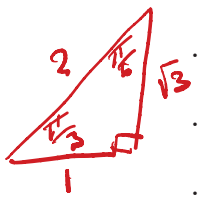
$t = 0 \quad D = 8 + 2\cos\left(\frac{4\pi}{25} \times 0\right)$

$= 10$ metres

b) What is the first time after 12 midnight that the water level will be at the depth of 7m? 2

$7 = 8 + 2\cos\left(\frac{4\pi t}{25}\right)$

$-\frac{1}{2} = \cos\left(\frac{4\pi t}{25}\right)$



Q2 & 3 $\quad \frac{2\pi}{3}, \frac{4\pi}{3}$

Period $= \frac{2\pi}{\frac{4\pi}{25}} = 12.5$ hrs

$\frac{2\pi}{3} = \frac{4\pi t}{25}$

$t = \frac{25 \times 2}{4 \times 3}$

$= \frac{25}{6}$ (1)

$= 4.16$

$\frac{5\pi}{3} = \frac{4\pi t}{25}$

$t = \frac{5 \times 25}{12} = 10.416$

first 7m after midnight

4hrs 10mins + 12hrs 30mins

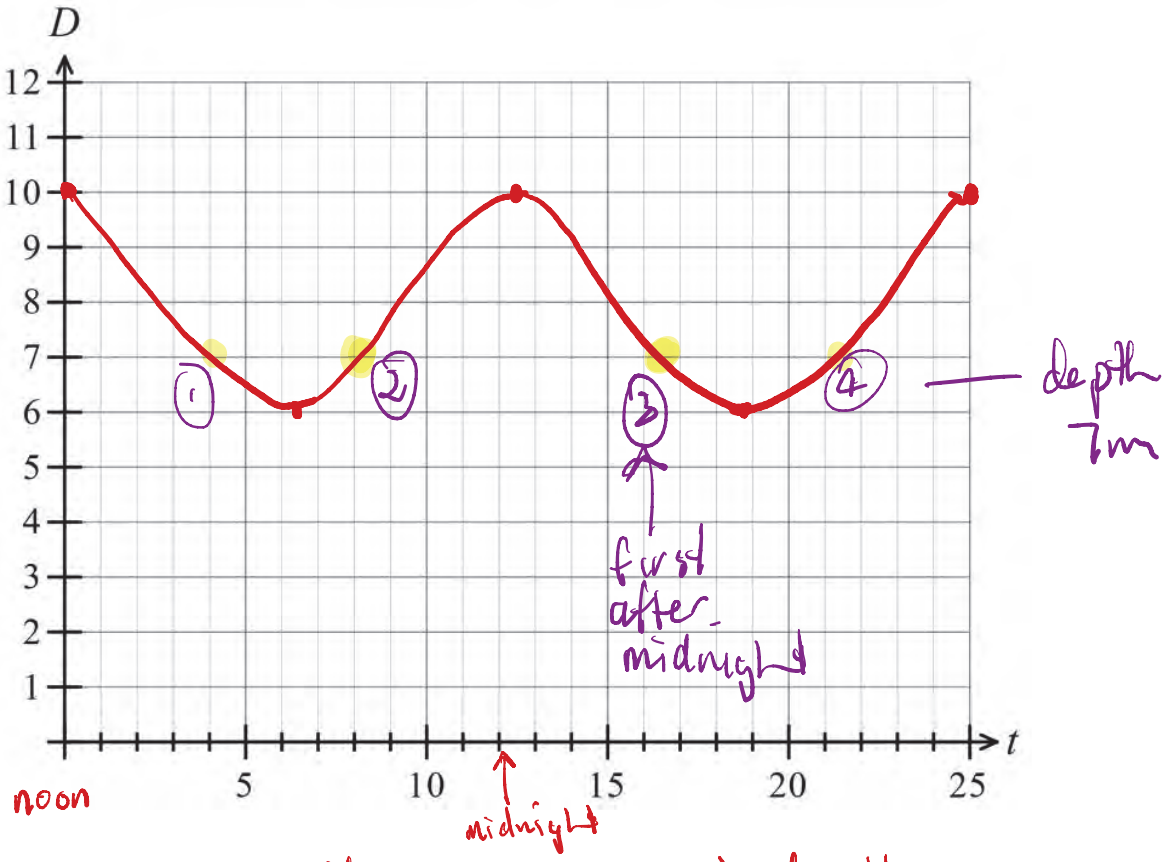
$=$ 16hrs 40mins (1)

$\therefore$ 4:40 pm

Question 31 continues on the next page.

c) Sketch the curve on the axes below, showing the maximum and minimum depths of the lagoon during the time period $0 \leq t \leq 25$. 2

Draw your sketch below:



max depth
= 10 m
at t = 0
t = 12.5
t = 25

min depth
= 8 - 2
= 6
at t = 6.25
& t = 18.75

Question 32 (5 marks)

A particle is moving along the x-axis such that at a time t seconds its displacement from the origin is x metres.

The acceleration of the particle is given by $a = \frac{8}{e^{2t}} + \frac{3}{e^{t}}$.

Initially its velocity is -6 m/s and its displacement is 5 m.

a) Find the equation for the displacement (x) of the particle in terms of t. 3

$a = \ddot{x} = 8e^{-2t} + 3e^{-t}$

$v = \dot{x} = -4e^{-2t} - 3e^{-t} + c$

at $t=0$ $\quad \dot{x} = -6$

$-6 = -4 - 3 + c$

$c = 1$

$\dot{x} = -4e^{-2t} - 3e^{-t} + 1$ (1)

$x = 2e^{-2t} + 3e^{-t} + t + c$

at $t=0$ $\quad x = 5$

$5 = 2 + 3 + 0 + c$

$c = 0$

$\therefore \quad x = 2e^{-2t} + 3e^{-t} + t$ (1)

$= \frac{2}{e^{2t}} + \frac{3}{e^{t}} + t$

(1) correct constants

Question 32 continues on the next page

b) At what time and where does the particle come to rest? 2

$-4e^{-2t} - 3e^{-t} + 1 = 0$

$\frac{4}{e^{2t}} + \frac{3}{e^{t}} - 1 = 0$

$4 + 3e^{t} - e^{2t} = 0$

Let $m = e^{t}$ $\quad 4 + 3m - m^2 = 0$

$(4 - m)(1 + m) = 0$ $\quad e^{t} \neq -1$ no soln's

$e^{t} = 4$

$t = \ln 4$ (1)

$x = \frac{2}{e^{2\ln 4}} + \frac{3}{e^{\ln 4}} + \ln 4$

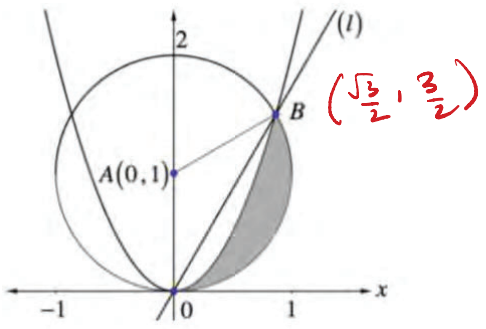
$= \frac{2}{16} + \frac{3}{4} + \ln 4$

$= \frac{7}{8} + \ln 4$ (1)

≈ 2.26 m

Question 33 (7 marks)

A circle centred at A with radius 1 unit intersects the parabola $y = 2x^2$ at the origin O and at the point B. The line l passes through O and B as shown in the diagram.



$(\frac{\sqrt{3}}{2}, \frac{3}{2})$

a) Show the coordinates of B are $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$. 2

eqn of circle $x^2 + (y-1)^2 = 1$ ①

parabola $y = 2x^2 \Rightarrow x^2 = \frac{y}{2}$ ②

$\frac{y}{2} + y^2 - 2y + 1 = 1$

$y^2 - \frac{3}{2}y = 0$

$y(y - \frac{3}{2}) = 0$

$y \neq 0$ $\quad y = \frac{3}{2}$ subst in ② $\quad x^2 = \frac{3}{2} \div 2$

$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$ but $x > 0$

$\therefore x = \frac{\sqrt{3}}{2}$

b) Find the angle OB makes with the positive x axis. 1

$m_{OB} = \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \frac{3}{2} \times \frac{2}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

$\therefore B = (\frac{\sqrt{3}}{2}, \frac{3}{2})$

$\tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$ $\Rightarrow$ $y = \sqrt{3}x$ eqn line

Question 33 continues on the next page

c) Show that $\angle OAB = \frac{2\pi}{3}$. 1

$\angle OAB = \frac{\pi}{2} - \frac{\pi}{3}$ (complementary adjacent $\angle$'s)

$\therefore \angle OAB = \frac{\pi}{6}$

$AO = AB = 1$ (equal radii)

$\angle AOB = \angle ABO = \frac{\pi}{6}$ (equal $\angle$'s opposite equal sides in isosceles $\triangle$)

$\angle OAB = \pi - \frac{\pi}{6} - \frac{\pi}{6}$

$= \frac{2\pi}{3}$

$r = a = b = 1$

d) Find the shaded area bounded by the circle and the parabola in the first quadrant as shown in the diagram. 3

Area of segment $= A_{sector} - A_{\triangle} = \frac{r^2\theta}{2} - \frac{ab\sin\theta}{2}$

$A_1 = \frac{r^2}{2}(\theta - \sin\theta)$

$= \frac{1}{2}\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ (1)

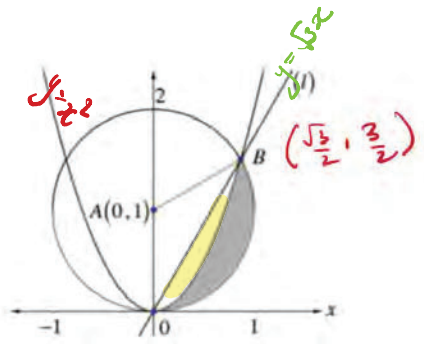
Area between $y = \sqrt{3}x$ and $y = 2x^2$

$A_2 = \int_0^{\frac{\sqrt{3}}{2}} (\sqrt{3}x - 2x^2)\,dx$

$= \left[\frac{\sqrt{3}}{2}x^2 - \frac{2}{3}x^3\right]_0^{\frac{\sqrt{3}}{2}}$

$= \frac{3\sqrt{3}}{8} - \frac{6\sqrt{3}}{24}$

$= \frac{\sqrt{3}}{8}$ units



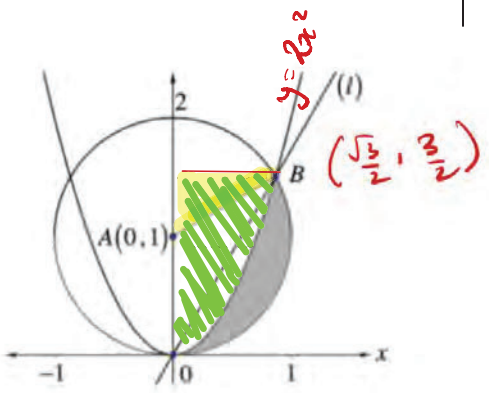
Shaded Area $= A_1 - A_2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\sqrt{3}}{8}\right)$

$= \frac{\pi}{3} - \frac{3\sqrt{3}}{8}$ units^2 (1)

End of paper

Section II Extra writing space

If you use this space, clearly indicate which question you are ai



Shaded = sector − A_1 + triangle

$A_{sector} = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 1^2 \times \frac{2\pi}{3}$

$= \frac{\pi}{3}$

$A_\Delta = \frac{1}{2} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{8}$

$y = 2x^2$

$\frac{y}{2} = x^2$

$x = \frac{\sqrt{y}}{\sqrt{2}}$

$A_1 = \frac{1}{\sqrt{2}} \int_0^{\frac{3}{2}} \sqrt{y}\, dy$ (area between parabola & y-axis).

$= \frac{1}{\sqrt{2}} \left[\frac{y^{3/2}}{3/2} \right]_0^{\frac{3}{2}}$

$= \frac{1}{\sqrt{2}} \times \frac{2}{3} \times \left[\sqrt{y^3} \right]^{3/2}$

$= \frac{2}{3\sqrt{2}} \times \sqrt{\frac{27}{8}}$

$= \frac{2}{3\sqrt{2}} \times \frac{3\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}}{2}$

Shaded = sector − A_1 + triangle

$= \frac{\pi}{3} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{8}$

$= \frac{\pi}{3} - \frac{4\sqrt{3}}{8} + \frac{\sqrt{3}}{8}$

$= \frac{\pi}{3} - \frac{3\sqrt{3}}{8}$

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.