Killara High School

Student Number

2024 Year 11 Preliminary Examination

Mathematics Advanced

09/09/2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (page 1 – 9)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - 90 marks (page 10 – 43)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room

This assessment task constitutes 40% of the course.

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. 135° converted to radians is:

A. $\frac{\pi}{4}$

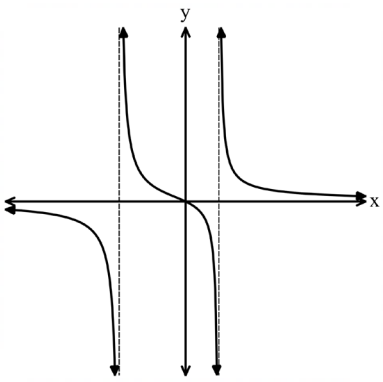
B. $\frac{3\pi}{4}$

C. π

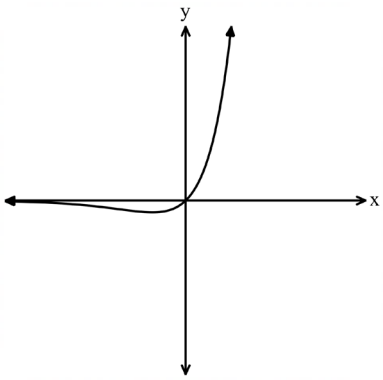
D. 135π

2. Which of the following functions is **not** continuous?

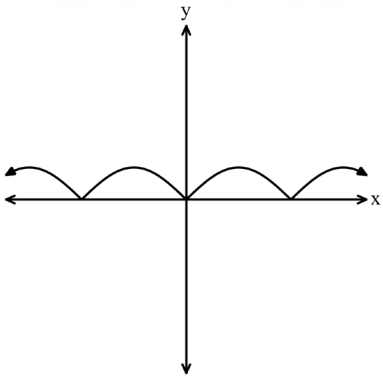
A.



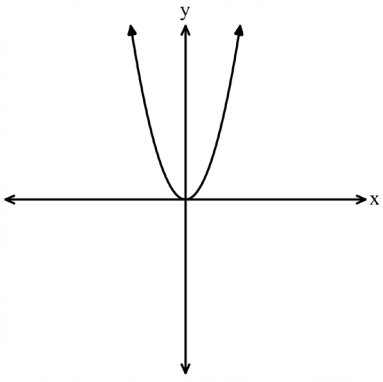
B.



C.



D.



3. Let ***X*** be the result when a spinner is spun.

The probability distribution of ***X*** is shown in the table below.

x	1	2	3	4
$P(X = x)$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$

Using the table above, calculate the value of $P(X \leq 3)$.

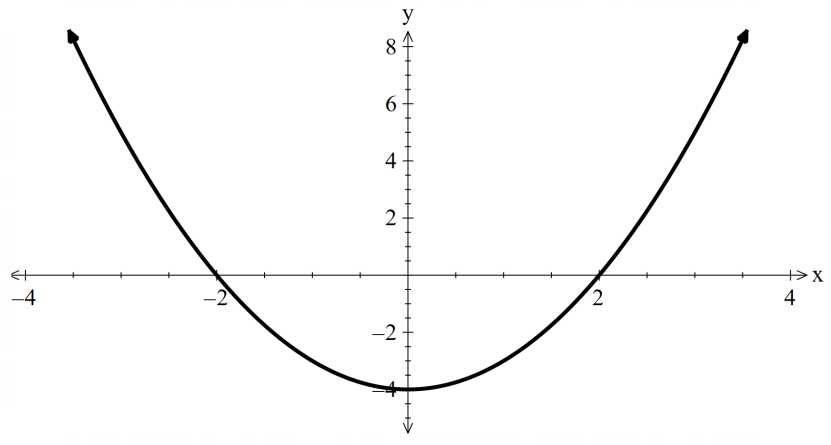
A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

4. Which statement is **false** for the function $y = x^2 - 4$?



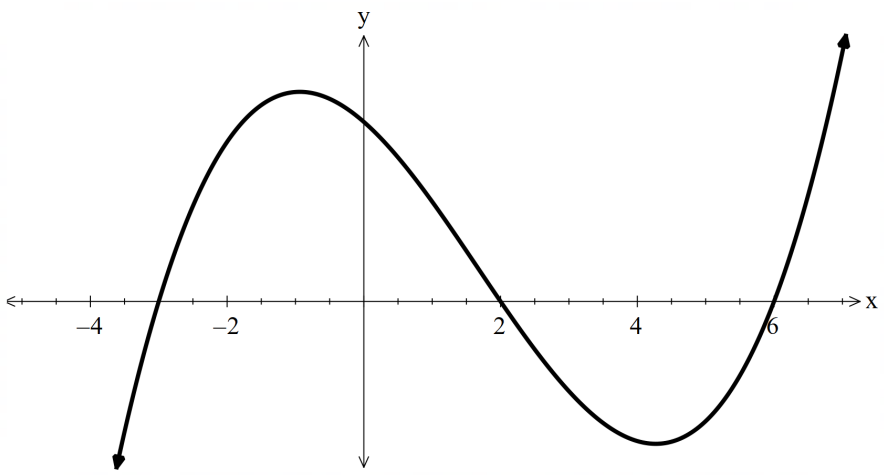
A. It is many-to-one function over the domain $(-\infty, \infty)$

B. It is one-to-one function over the domain $(-\infty, \infty)$

C. It is one-to-one function if the domain is restricted to $(0, \infty)$

D. It is one-to-one function if the domain is restricted to $(-\infty, 0)$

5. Determine the equation that best represents the polynomial shown below.



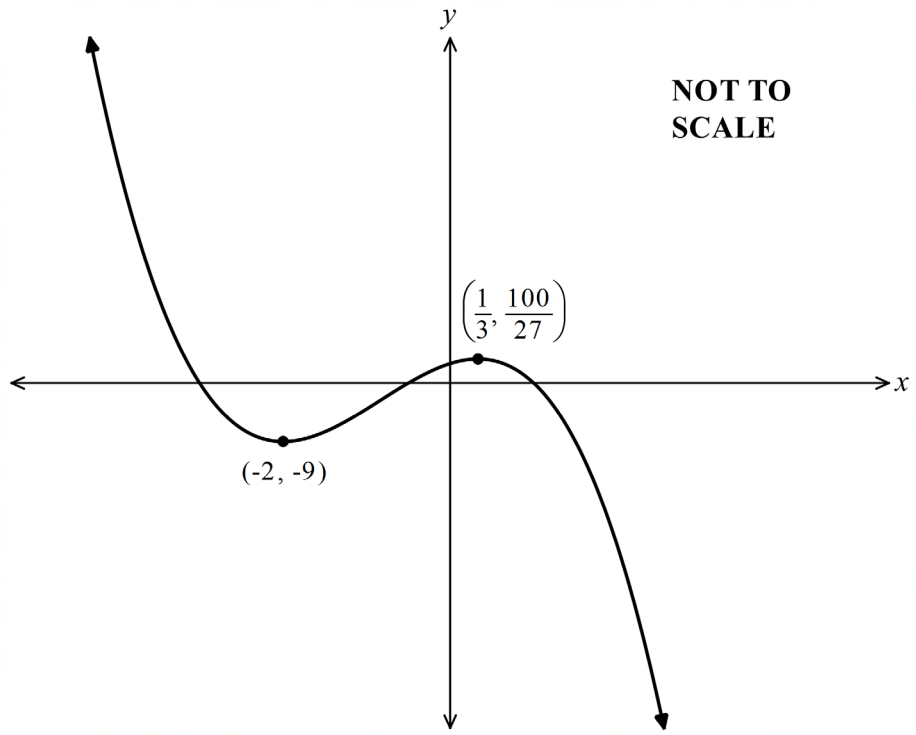
A. $y = -(x - 3)(x + 2)(x + 6)$

B. $y = -(x + 3)(x - 2)(x - 6)$

C. $y = (x - 3)(x + 2)(x + 6)$

D. $y = (x + 3)(x - 2)(x - 6)$

6. The graph of a function $y = f(x)$ has turning points at $\left(\frac{1}{3}, \frac{100}{27}\right)$ and $(-2, -9)$ as shown below.



For what values of x is $f'(x) < 0$?

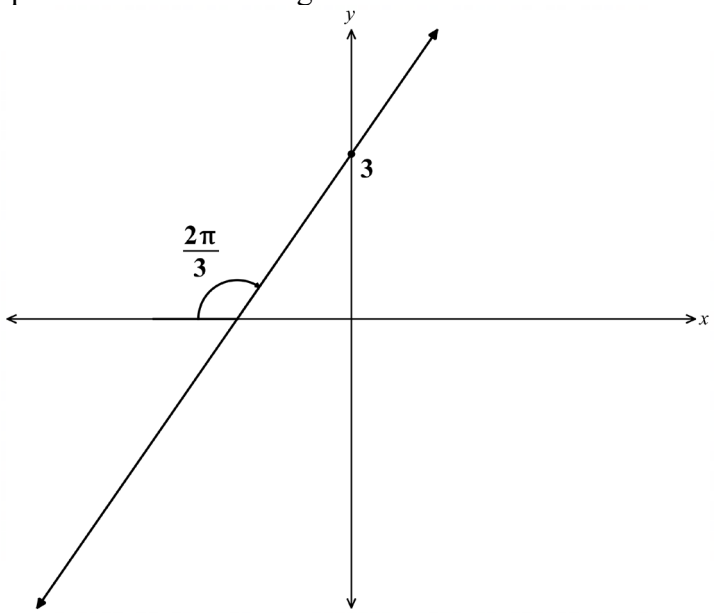
A. $x \in (-2, 0) \cup \left(\frac{1}{3}, \infty\right)$

B. $x \in \left(-9, \frac{100}{27}\right)$

C. $x \in (-\infty, -2) \cup \left(\frac{1}{3}, \infty\right)$

D. $x \in (-\infty, -2] \cup \left(\frac{1}{3}, \infty\right)$

7. Determine the equation of the following line.



A. $y = \frac{2\pi}{3}x + 3$

B. $y = -\frac{2\pi}{3}x + 3$

C. $y = -\sqrt{3}\,x + 3$

D. $y = \sqrt{3}\,x + 3$

8. If $\cot\theta = \frac{24}{7}$ and $\pi < \theta \leq \frac{3\pi}{2}$. Find the exact value of $\sin\theta$.

A. $-\frac{25}{24}$

B. $-\frac{7}{25}$

C. $\frac{7}{25}$

D. $\frac{24}{25}$

9. If $\boldsymbol{X}$ is a discrete random variable such that $P(X > 5) = a$ and $P(X > 8) = b$.

Find an expression that represents $P(X \leq 5 | X \leq 8)$ in terms of a and b.

A. $\dfrac{a}{b}$

B. $\dfrac{ab}{1-b}$

C. $\dfrac{1-a}{1-b}$

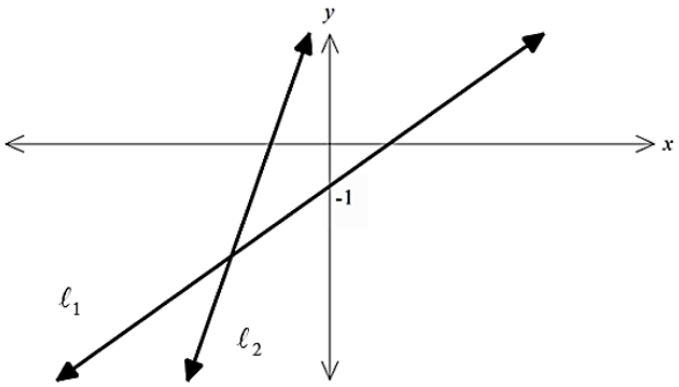
D. $\dfrac{a-b}{1-b}$

10. The straight lines shown below l_1 and l_2 have equations $x + ay = b$ and $x + cy = d$ respectively. Which of the following are true?

I. $a < c$

II. $b > d$

III. $a + b < c + d$



A. I and II

B. I and III

C. II and III

D. I, II and III

End of Section I

Killara High School

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2024 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 1

09/09/2024

Section II

90 Marks
Attempt Questions 11 - 34
Allow about 2 hours and 45 minutes for this section.

Booklet I – Attempt Questions 11 – 27 (55 marks)
Booklet II – Attempt Questions 28 – 34 (35 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 27 - 28
- If you use this space, clearly indicate which question you are answering.

Q	Marks
11	/2
12	/2
13	/4
14	/2
15	/2
16	/4
17	/2
18	/5
19	/4
20	/2
21	/3
22	/5
23	/6
24	/2
25	/2
26	/4
27	/4
Total	**/55**

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Section II

In Questions 11 - 27, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Fully simplify the following expression **2**

$$\frac{a^2 - 3a}{2ax} \times \frac{4a^2x}{2ax - 6x}$$

Question 12 (2 marks)

Fully simplify the following expression using index laws. Leave your answer in the simplest form. **2**

$$\frac{\left(x^2 y^{\frac{1}{2}}\right)^2}{y\sqrt{x^3}}$$

Question 13 (4 marks)

Solve the following trigonometric equations, for $0 \leq \theta \leq 2\pi$.

(a) $\sin\theta = -\frac{1}{\sqrt{2}}$ **2**

(b) $\sec\theta = \frac{2}{\sqrt{3}}$ **2**

Question 14 (2 marks)

Solve: **2**

$$|5 - 3x| = 6$$

Question 15 (2 marks)

Evaluate: **2**

$$\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$$

Question 16 (4 marks)

The productivity P, of a student is directly proportional to the square of the amount of hours slept in the night before, S.

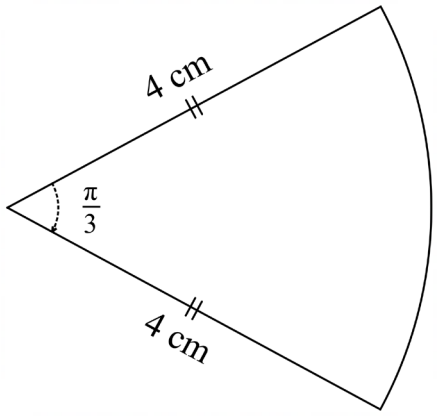
Sam stayed up late the night before school and as a result, he only had 4 hours of sleep. His productivity the next day was 2 hours.

(a) Calculate the constant of variation. **2**

(b) If Sam wants to be productive for 8 hours in a day, how many hours of sleep will he need in the night before? **2**

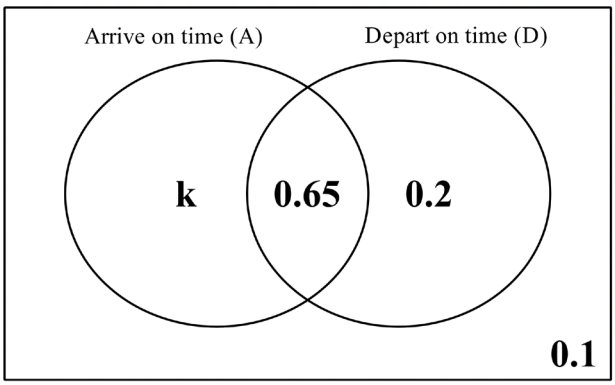
Question 17 (2 marks)

The sector shown below has a radius of 4 cm and the angle subtended at the centre is $\frac{\pi}{3}$ radians. Calculate the exact perimeter of this sector. **2**



Question 18 (5 marks)

At Sydney International Airport, the probability that all passengers arrive on time and the probability that a flight departs on time are represented on the Venn diagram below.



Where $P(A) = 0.7$ and $P(D) = 0.85$.

(a) Show that the value of $k = 0.05$. **1**

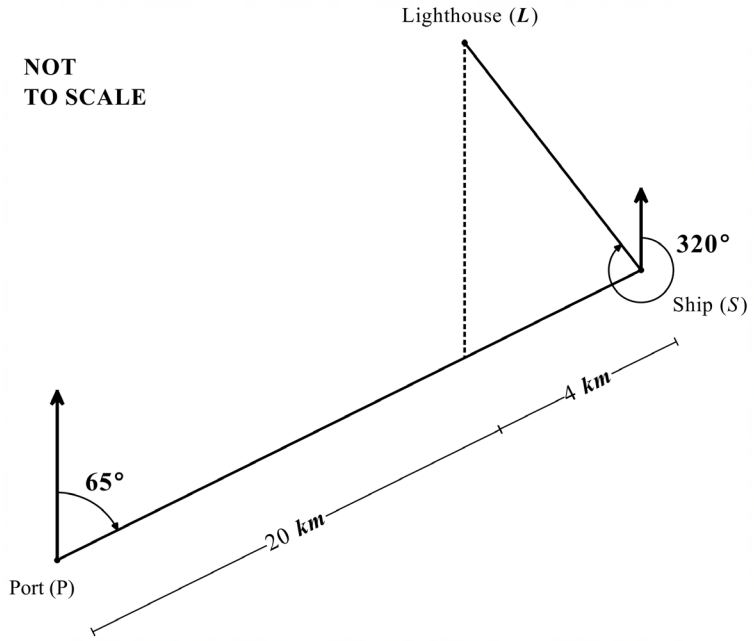
(b) Given that all passengers for a flight arrive on time, find the probability that the flight does **not** depart on time. **2**

(c) Are the events of passengers arriving on time and flights departing on time independent? Justify your answer with the relevant calculations. **2**

Question 19 (4 marks)

A ship (S) leaves a port (P) travelling on a bearing of 065°. After travelling 20 kilometres, the ship is due south of a lighthouse.

The ship continues on this bearing for a further 4 kilometres, then finds the bearing of the lighthouse to be 320°.



(a) Show that $\angle PSL$ is 75°. **2**

(b) Calculate the distance from the ship to the lighthouse at this time to the nearest kilometre. **2**

Question 20 (2 marks)

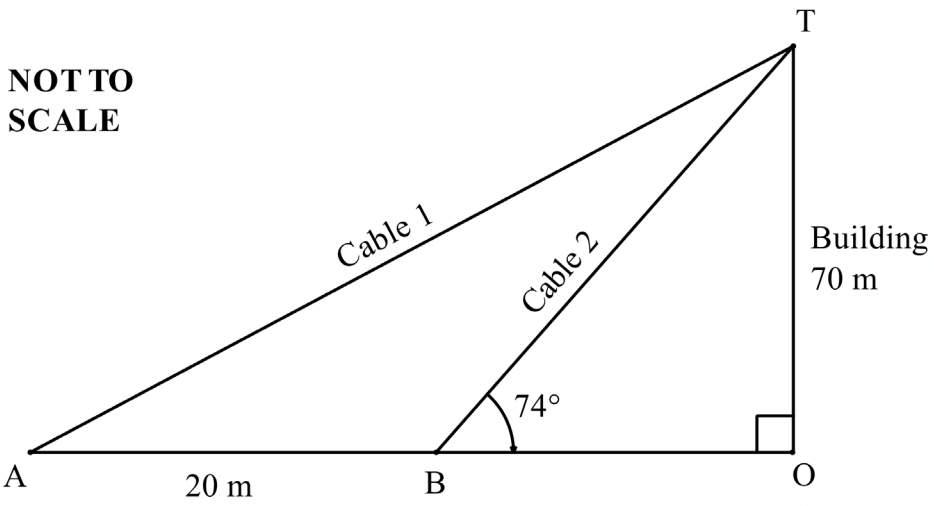
For what values of k does the following parabola have real and distinct roots? **2**

$$x^2 - 5x + (6 - k) = 0$$

Question 21 (3 marks)

Two support cables have been secured from point A and point B to the top of a building that is 70 metres in height, and the distance between A and B is 20 metres. **3**

From point B, the angle of elevation to the top of the building is 74°.



Find the length of cable 1 to the nearest 2 decimal places.

Question 22 (5 marks)

Let $f(x) = \sqrt{x}$ and $g(x) = x^4 + 1$.

(a) Let $h(x) = f(g(x))$. State the domain and range of $h(x)$. **3**

(b) State whether $h(x)$ is odd, even or neither. Justify your answer with the appropriate calculations. **2**

Question 23 (6 marks)

Differentiate the following functions.

(a) $f(x) = 3x^4 + 7x^3 - 6x^2 + 5$ **1**

(b) $f(x) = \sqrt{3x - x^2}$ **2**

(c) $f(x) = x^3(4 - x)^5$ **3**

Question 24 (2 marks)

Prove the following trigonometric identity. **2**

$$\frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)} = 2\tan(x)\sec(x)$$

Question 25 (2 marks)

Determine the transformation that has been applied to $x^2 + y^2 = 9$ to result in $x^2 - 4x + y^2 + 6y + 4 = 0$. Justify your answer with the relevant calculations. **2**

Question 26 (4 marks)

Given that

$$f(x) = \frac{\sqrt{x}}{x+1}$$

(a) Show that **3**

$$f'(x) = -\frac{x-1}{2\sqrt{x}(x+1)^2}$$

(b) Hence, or otherwise, find the value of x such that $f'(x) = 0$. **1**

Question 27 (4 marks)

When Mary's mother goes shopping, the probability that Mary will go with her is 40%.

If Mary goes shopping with her mother, her mother will buy ice cream for her 70% of the time.

When Mary does not go shopping with her mother, her mother will buy ice cream for her 30% of the time.

(a) Calculate the probability that Mary's mother will buy ice cream for her. **2**

(b) Calculate the probability that Mary went shopping with her mother, given that her mother bought her ice cream. **2**

End of Booklet I

Proceed to Booklet II

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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2024 Year 11 Preliminary Examination

Mathematics Advanced

Section II Answer Booklet 2

09/09/2024

Q	Marks
28	/2
29	/8
30	/6
31	/4
32	/5
33	/7
34	/3
Total	**/35**

Booklet 2 – Attempt Questions 28 – 34 (35 marks)

Instructions

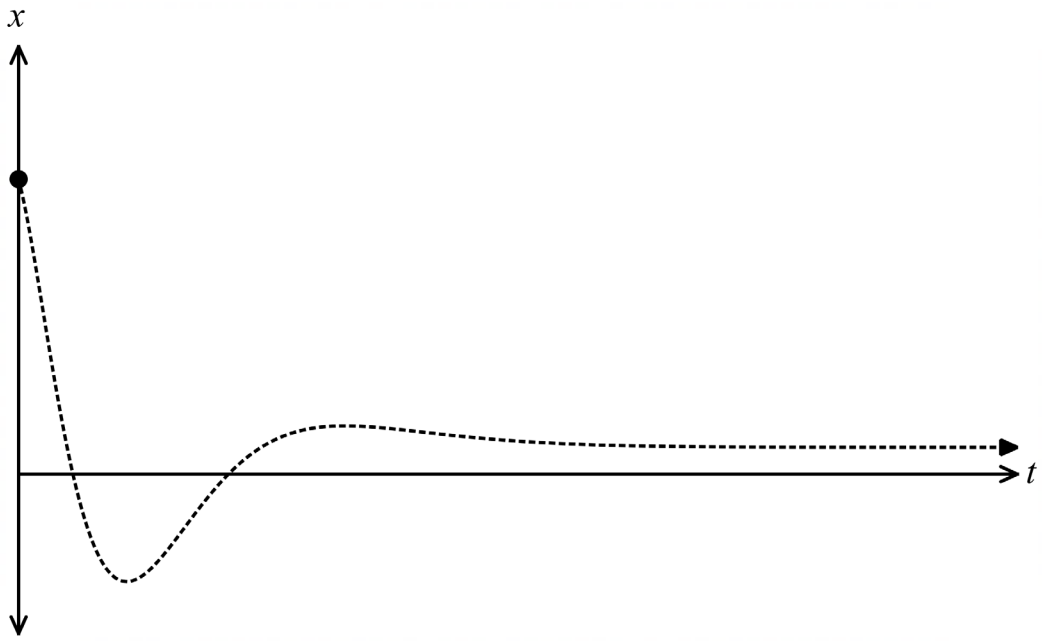
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 41 - 42.
- If you use this space, clearly indicate which question you are answering.

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Question 28 (2 marks)

A particle is moving along a straight line and its displacement graph is modelled on the axis below. It is given that the particle's initial velocity is -1. **2**

Sketch the corresponding velocity function on a sperate axis below.





Question 29 (8 marks)

Water is leaking from the bottom of a tank. It is given that V is the volume of water in litres remaining in the tank after t minutes and $0 \leq t \leq 25$.

$$V(t) = 500\left(1 - \frac{1}{25}t\right)^3$$

(a) What is the initial volume of water in the tank? **1**

(b) What is the average rate at which water is leaking in the first 10 minutes? **2**

(c) Show that the initial rate of water leaking from the tank is 60 L/min. **2**

Question 29 continues on page 33

Question 29 (continued)

(d) Hence, calculate the time at which water is leaking at half the initial rate, correct to the nearest whole number. **3**

End of Question 29

Question 30 (6 marks)

The following table shows the probability distribution of a discrete random variable X.

x	1	2	3	4
$P(X = x)$	k^2	$7k + 2$	$-2k$	$3k^2$

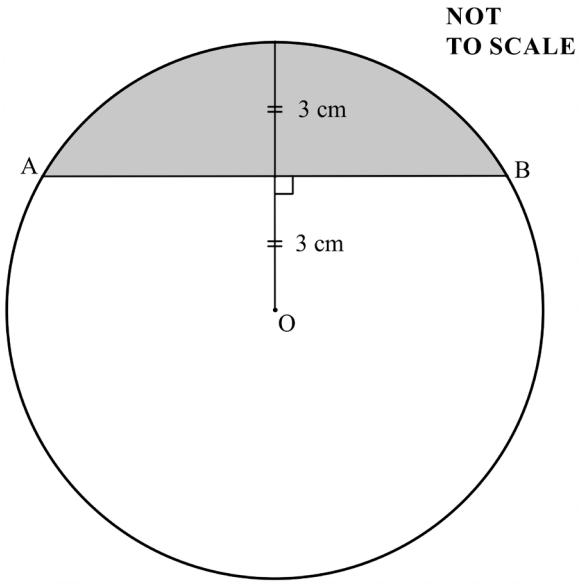
(a) Show that $k = -\frac{1}{4}$. **3**

(b) Hence, or otherwise calculate the $E(X)$. **1**

(c) Hence, or otherwise calculate $\text{Var}(X)$ to the nearest 2 decimal places. **2**

Question 31 (4 marks)

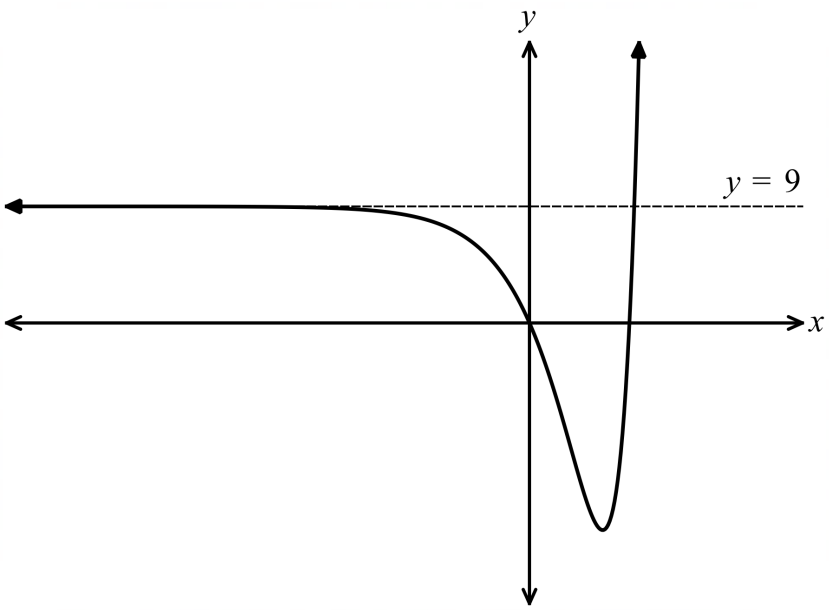
A circle has radius 6 cm and centre O. A chord AB has been drawn such that it bisects the radius as shown. **4**



Find the exact area of the shaded minor segment.

Question 32 (5 marks)

The graph of $f(x) = 3^{2x} - 10(3^x) + 9$ is shown below.

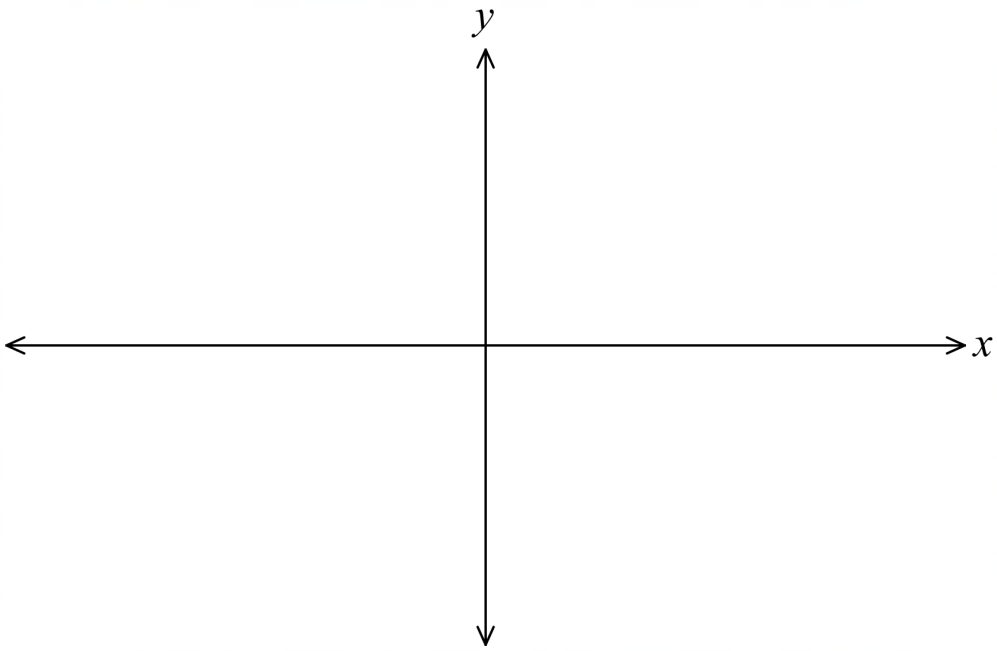


(a) Find the x-intercepts using an algebraic method. **3**

Question 32 continues on page 37

Question 32 (continued)

(b) On the axis below, sketch the graph of $y = f(-x)$. Labelling the intercepts and the asymptote. **2**



End of Question 32

Question 33 (7 marks)

A soccer arena has a maximum capacity of 15000 spectators. The number of spectators, N, and the ticket price in dollars P, are modelled using the relationship

$$N = m \times P + c,$$

where m and c are constants.

The following information is known:

- When the price of a ticket is \$50, the attendance averages 12500 spectators.
- When the price of the ticket is \$35, the attendance averages 14000 spectators.

(a) By developing and solving a pair of simultaneous equations, show that the number of spectators as a function of ticket price can be expressed as **3**

$$N = -100P + 17500$$

Question 33 continues on page 39

Question 33 (continued)

(b) The revenue generated from an event, R, is calculated using **2**

$$R = N \times P$$

What is the maximum revenue generated from the event?

(c) The manager of this event wants the arena to be filled with 15000 spectators. Show that this will result in a reduction of revenue by \$390 625 from the maximum revenue. **2**

End of Question 33

Question 34 (3 marks)

Solve for $\boldsymbol{y}$ in the following quadratic equation for $0° \leq y \leq 360°$ using trigonometric identities. **3**

$$\sin^2 y + \sqrt{3}\sin y + 1 - \cos^2 y = 0$$

End of Examination

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.