Student's Name: _____

Student Number: | | | | | | | | | |

Teacher's Name: _____

ABBOTSLEIGH

2024
Year 11 Preliminary
Assessment Task 3

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- NESA approved calculators may be used.
- NESA approved reference sheet is provided.
- All necessary working should be shown in every question.
- Answer the Multiple Choice questions on the answer sheet provided.
- Answer Questions 11-33 in the spaces provided.
- Extra writing spaces are provided at the end of each section.
- In Questions 11-33, show relevant mathematical reasoning and/or calculations

Total marks – 80

- Attempt Sections I to VII

Section I Pages 3–8

10 marks

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Sections II-VII Pages 9 - 38

70 marks

- Attempt Questions 11–33.
- Allow about 1 hour and 45 minutes for this section.

Outcomes to be assessed:

Advanced Mathematics

Preliminary:

A student:

MA11-1	uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems.
MA11-2	uses the concepts of functions and relations to model, analyse and solve practical problems.
MA11-3	uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes.
MS11-4	uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities.
MS11-5	interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems.
MS11-6	manipulates and solves expressions using the logarithmic and index laws, and uses logarithms and exponential functions to solve practical problems.
MS11-8	uses appropriate technology to investigate, organise, model and interpret information in a range of contexts
MS11-9	provides reasoning to support conclusions which are appropriate to the context.

SECTION I

10 marks

Attempt Questions 1–10
Use the multiple choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.
Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

(A) ○ (B) ● (C) ○ (D) ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

(A) ● (B) ⊗ (C) ○ (D) ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct ←

(A) ⊗ (B) ⊗ (C) ○ (D) ○

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1. If $f(x) = 2x + 1$, what is the value of $f(f(1))$?

A. 1

B. 3

C. 5

D. 7

2. Which if the following quadratic equations has 2 distinct real and rational roots?

A. $y = -x^2 - 3x + 5$

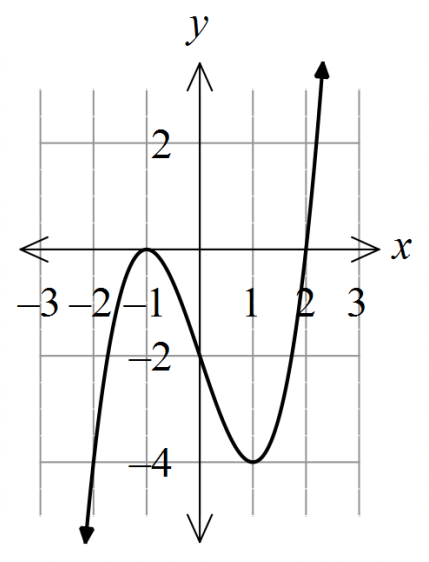
B. $y = 4x^2 - x - 5$

C. $y = -x^2 - 4$

D. $y = 2x^2$

Section I (continued)

3. Which of the following equations best describes the graph below?



A. $y=(x+1)^2(x-2)$

B. $y=(x+1)(x-2)^2$

C. $y=(x-1)^2(x+2)$

D. $y=(x-1)(x+2)^2$

4. Which of the following is a many-to-one relation?

A. $x^2+y^2=4$

B. $x=y^2+4$

C. $y=|x-4|$

D. $y=4e^x$

Section I (continued)

5. Let $f(x) = x^2 + ax,$ where a is constant and $f(10) = f(2).$
Which of the following statements is true?

A. $f\left(\frac{1}{2}\right) < f(1) < f(3)$

B. $f(1) < f\left(\frac{1}{2}\right) < f(3)$

C. $f(1) < f(3) < f\left(\frac{1}{2}\right)$

D. $f(3) < f(1) < f\left(\frac{1}{2}\right)$

6. What is the number of solutions of the equation $\left|2\cos x + 1\right| = 1$ for $0 \le x \le 2\pi$?

A. 1

B. 2

C. 3

D. 4

7. What is the domain of the function $f(x) = \ln(x-1) + \sqrt{2-x}$?

A. $x \in (1, 2)$

B. $x \in (1, 2]$

C. $x \in [1, 2)$

D. $x \in [1, 2]$

8. If $N = 10e^{kt}$ and $N = x$ when $t = 1$ what is the value of N when $t = 2$?

A. $\frac{1}{100}x^2$

B. $\frac{1}{10}x^2$

C. $10x^2$

D. $100x^2$

9. If $x = \log_e(1+y) - \log_e(1-y),$ where $-1 < y < 1,$ which of the following is correct?

A. $y = \frac{1}{2}e^x$

B. $y = \frac{x-1}{x+1}$

C. $y = \frac{e^x - 1}{e^x + 1}$

D. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

10. Consider a curve with the following properties:

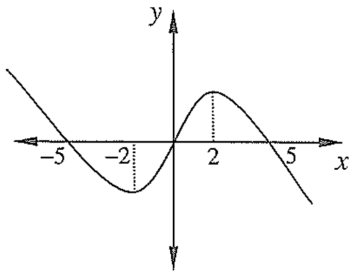
$g(x)$ is odd.

$g(5) = 0$ and $g'(2) = 0$.

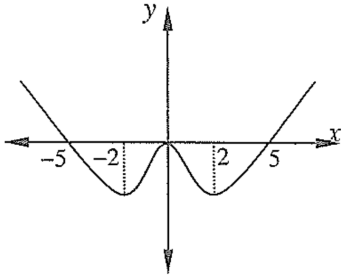
$g'(x) > 0$ for $x > 2$.

Which of the following could be the graph of $y = g(x)$?

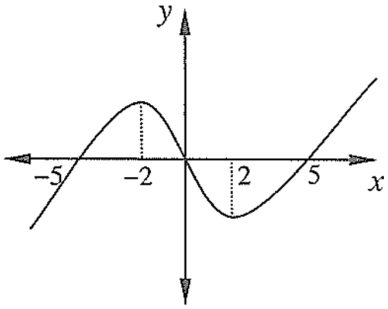
A.



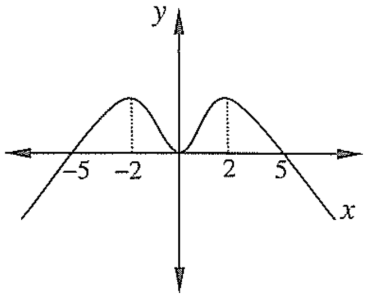
B.



C.



D.



End of Section I

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Student Number: | | | | | | | | | |

Sections II - VII

70 marks
Attempt Questions 11-33
Allow about 1 hour and 45 minutes for this section

Answer the questions in the spaces provided.

These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 13, 18, 23, 28, 32, 38. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks) **Marks**

Solve the following equation: **2**

$$|5-3x|=11$$

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Section II continues on page 10

Question 12 (2 marks) **Marks**

Given that $a = 4$ and $b = \sqrt{3}$, evaluate $\dfrac{a+b^2}{ab}$. **2**

Leave your answer with a rational denominator.

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Question 13 (2 marks)

Simplify fully $\dfrac{5}{x^2-3x+2} - \dfrac{3}{x-1}$. **2**

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Section II continues on page 11

Question 14 (2 marks) **Marks**

Given that $\tan x = 2$ and $\cos x < 0$, find the exact value of $\sin x$. **2**

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Question 15 (2 marks)

The probability that Gemma gets a concert booking with her band on any given weekend is 0.65. **2**

What is the probability that over two consecutive weekends she will get at least one concert booking?

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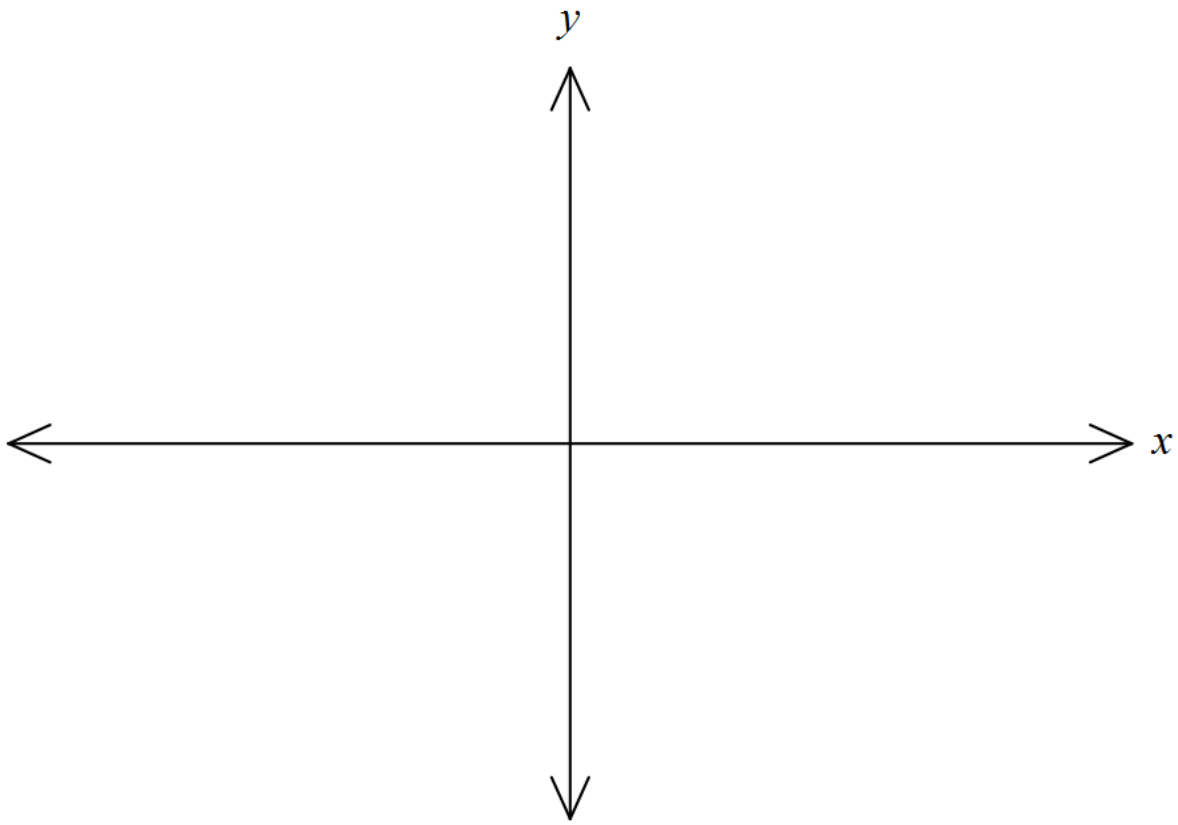
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Section II continues on page 12

Question 16 (2 marks) **Marks**

Sketch $y = e^{-x} - 2$. **2**

Show all intercepts and asymptotes.



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End of Section II

Section II Extra writing space

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Question Number: |

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Section III (11 marks)

Question 17 (4 Marks) **Marks**

Differentiate the following with respect to x. Leave all answers without fractional indices:

(a) $\dfrac{7}{x^3} - \dfrac{7}{\sqrt[3]{x}}$. **2**

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(b) $\sqrt{(2-3x)^5}$. **2**

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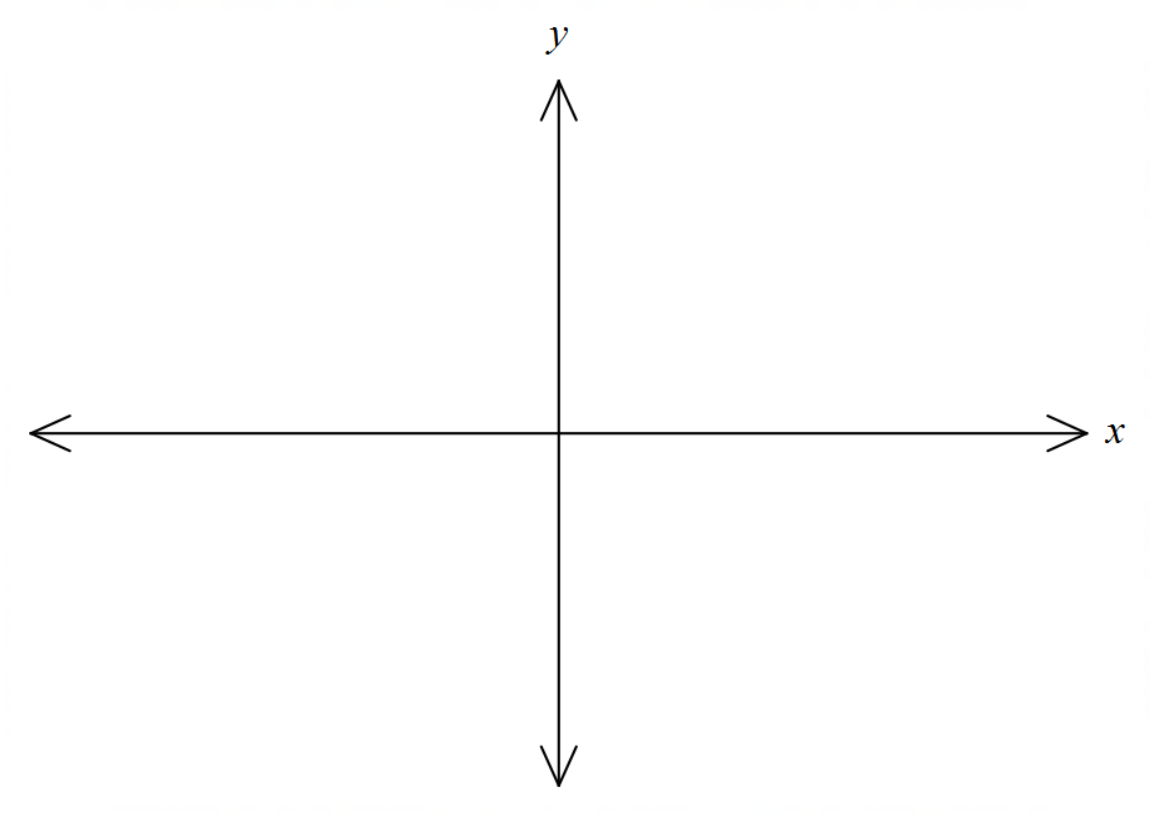
Section III continues on page 16

Question 18 (4 Marks) **Marks**

A function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} 2^x & \text{for } x>1 \\ \sqrt{1-x^2} & \text{for } -1\le x\le \\ -(2+x) & \text{for } x<-1 \end{cases}$$

(a) Draw the graph of $y=f(x)$. **3**



(b) Explain why this is no longer a function if the third part of the definition is changed to $-(2+x)$ for $x\le 1$. **1**

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Section III continues on page 17

Question 19 (3 marks) **Marks**

At time t years after it was purchased the value $\$V$ of a car is given by $V = 40000e^{-0.4t}$. **3**

Find in years and months, correct to the nearest month, the time taken for the value of the car to fall to \$1000.

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End of Section III

Section III Extra writing space

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Question Number: |

Student Number: | | | | | | | | | |

Section IV (11 marks)

Question 20 (2 marks) **Marks**

Evaluate: **2**

$$\frac{25^x \times 3^{2x+5}}{15^{2x+1}}.$$

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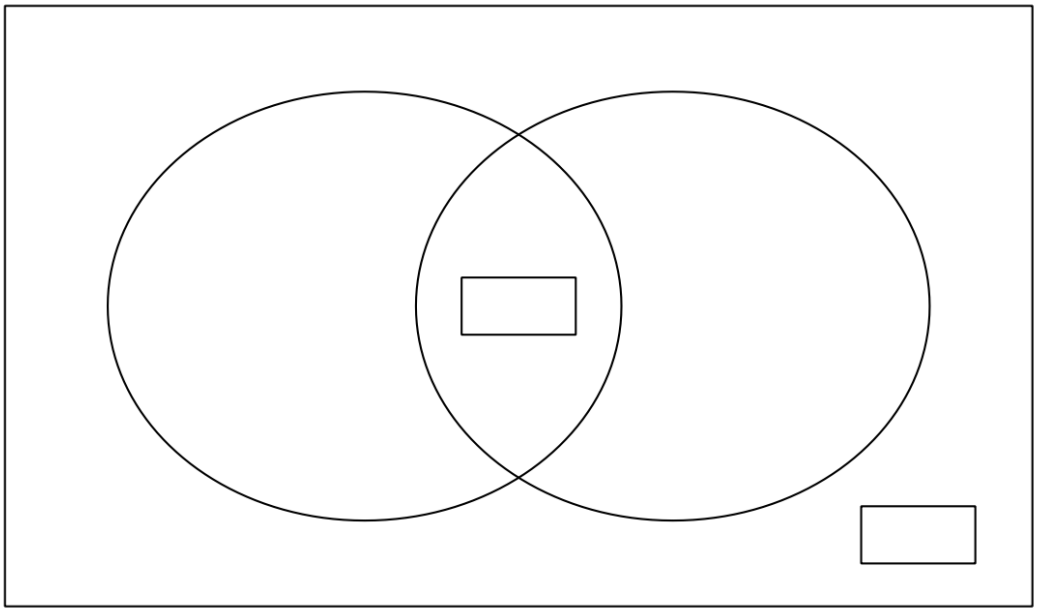
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Section IV continues on page 20

Question 21 (3 Marks) **Marks**

A class has 28 students. 5 of them study neither Art nor Music and 8 of them study exactly one of Art and Music. Two students are chosen at random from the class.

(a) Draw a Venn diagram to represent this information showing **only** the number studying both Art and Music, and the number studying neither Art nor Music. **1**



(b) Find in simplest fraction form the probability that both students chosen study both Art and Music. **2**

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Section IV continues on page 21

Question 22 (3 marks) **Marks**

Sketch the function $f(x) = x^2 - 6x + 10$ for the domain $[-1, 3)$. **3**

Show all important features including intercepts and vertex.



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Section IV continues on page 22

Question 23 (3 marks) **Marks**

Find the coordinates of the points where the tangent to the curve $y = x^2e^x$ is parallel to the x-axis. **3**

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End of Section IV

Section IV Extra writing space

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Question Number: |

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Section V (10 marks)

Question 24 (2 marks) **Marks**

A triangle has sides of length 5 cm, 7 cm and 8 cm. **2**

Find the size of the smallest angle, correct to the nearest minute.

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Question 25 (2 marks)

Solve $x^2 - 6x - 6 = 0$ by completing the square. Leave your answer in exact form. **2**

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Section V continues on page 26

Question 26 (4 marks) **Marks**

(a) Show that the tangent to the curve $y = x^2 + 1$ at the point $P\left(a,\ a^2 + 1\right)$ has equation $2ax - y - a^2 + 1 = 0$. **2**

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(b) Find the coordinates of any points on the curve where the tangent to the curve passes through the origin. **2**

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Section V continues on page 27

Question 27 (2 marks) **Marks**

Consider the following function: **2**

$$f(x)=\begin{cases} x^2, & x<0 \\ 2x-3, & x\geq 0 \end{cases}$$

Find the size of the value of x if $f(x)=25$.

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End of Section V

Section V Extra writing space

Student Number:

Question Number:

Student Number: | | | | | | | | | |

Section VI (9 marks)

Question 28 (4 Marks) **Marks**

(a) Show that: **1**

$$\tan^2 A\left(1-\sin^2 A\right)=\sin^2 A.$$

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(b) Hence, or otherwise, solve: **3**

$$4\tan^2 A\left(1-\sin^2 A\right)=1,\ \text{for}\ \ 0\le A\le 2\pi.$$

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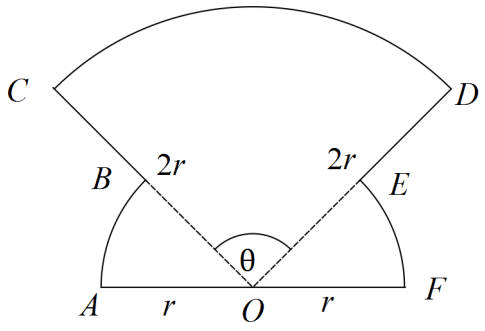
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Section VI continues on page 30

Question 29 **Marks**



NOT TO SCALE

The shape *OABCDEFO* shown above is a design for a logo.

In the design:

- *OAB* is a sector of a circle centre *O* and radius *r*
- Sector *OFE* is congruent to sector *OAB*
- *ODC* is a sector of a circle centre *O* and radius 2*r*
- *AOF* is a straight line

Given that the size of the angle *COD* is θ radians,

(a) Show that $\angle AOB = \dfrac{\pi - \theta}{2}$. **1**

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Section VI continues on page 31

Question 29 (continued)

(b) Show that the area of the logo is **2**

$$\frac{1}{2}r^2\left(3\theta+\pi\right).$$

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(c) Find the perimeter of the logo, giving your answer in simplest form in terms of r, θ and π. **2**

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End of Section VI

Section VI Extra writing space

Student Number: | | | | | | | | | |

Question Number: |

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Section VII (17 marks)

Question 30 (3 marks) **Marks**

Solve the equation: **3**

$$\log_2 x - \frac{3}{\log_2 x} = 2.$$

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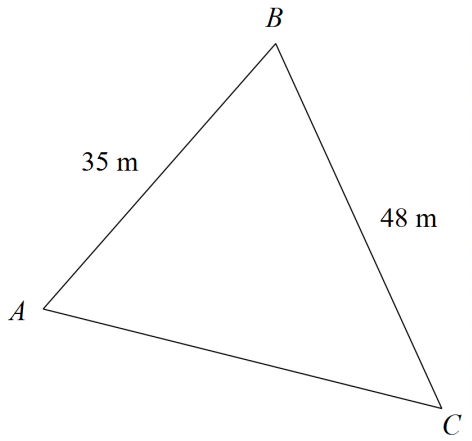
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Section VII continues on page 34

Question 31 (4 marks) **Marks**

Three girls are standing on the school oval. A is 35 metres from B and B is 48 metres from C.

The bearing of B from A is 036°T and the bearing of C from B is 156°T.



NOT TO SCALE

(a) Show all the information on the diagram above. **1**

(b) Show that $\angle ABC = 60^{\circ}$. **1**

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(c) Show that the distance of C from A is 43 metres. **2**

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Section VII continues on page 35

Question 31 (continued)

(d) Hence, or otherwise, find the bearing of A from C. **3**

Give your answer correct to the nearest degree.

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Section VII continues on page 36

Question 32 (4 marks) **Marks**

In a local netball competition, a team can either win or lose a match.

A certain team has a probability of 0.45 of winning a match if the coach's daughter is playing in that match or a probability of 0.75 if she is not. The probability of the coach's daughter playing is 0.4.

(a) Find the probability that the team will win a match. **2**

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(b) Given that the team has won a match, find the probability that the coach's daughter was playing on that day. **2**

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Section VII continues on page 37

Question 33 (3 marks) **Marks**

Given that $y = \dfrac{x-4}{2+\sqrt{x}}$, $x > 0$ find the value of A such that $\dfrac{dy}{dx} = \dfrac{1}{A\sqrt{x}}$. **3**

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End of Paper

Section VII Extra writing space

Student Number:

Question Number: