HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 11 Yearly Examination
Assessment 3 Term 3 2022

STUDENT NUMBER: _____

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Students are permitted to refer to one double-sided A4 page of handwritten topic summaries which they prepared prior to this assessment

Total Marks: 81

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 71 marks (pages 8–24)

- Attempt Questions 11–26
- Allow about 1 hour and 45 minutes for this section

Section I

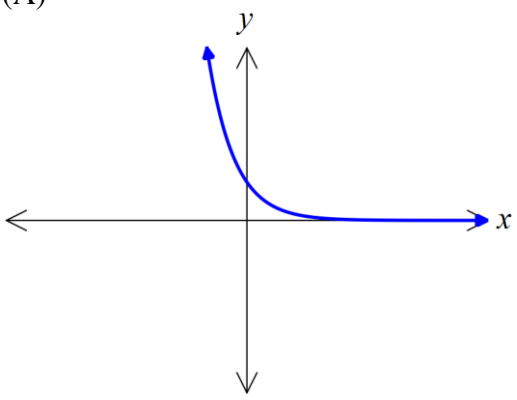
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 5

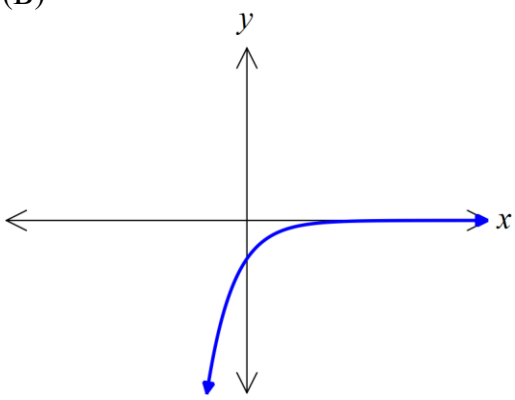
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1 Which of the following best represents the graph of $y = 5(0.2)^x$?

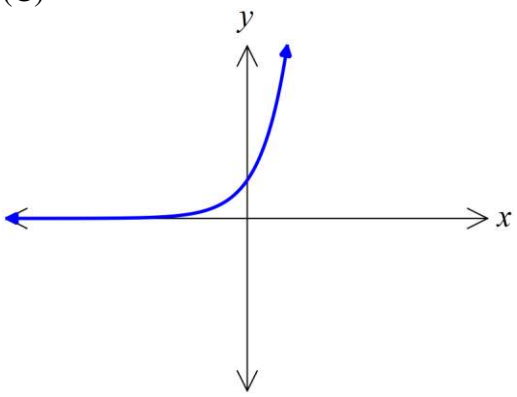
(A)



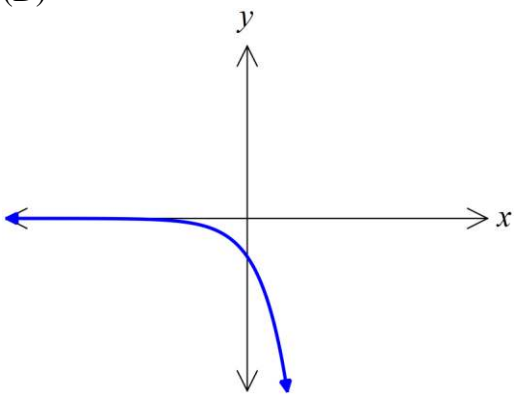
(B)



(C)



(D)



2 The tangent to the curve $y = x^2$ at the point $(2,4)$ makes the angle θ with the positive direction of the x-axis. What is the value of θ, correct to the nearest minute?

(A) 1°19'

(B) 1°20'

(C) 75°57'

(D) 75°58'

3 Which expression is a simplification of $\dfrac{\sin(\pi - x)}{\sin\left(\dfrac{\pi}{2} - x\right)}$?

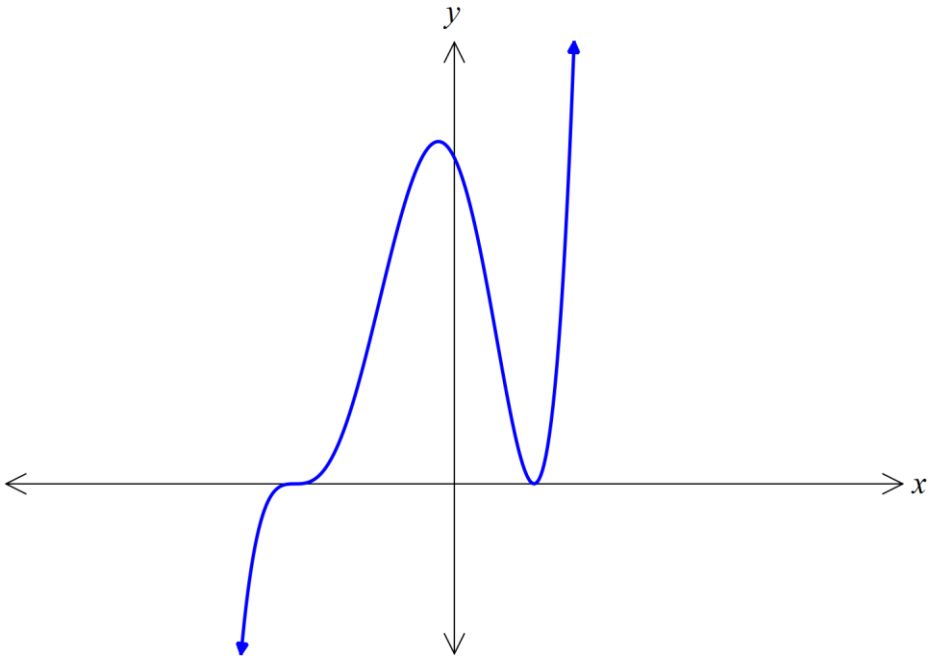
(A) $\dfrac{-\sin x}{1-\sin x}$

(B) $-\tan x$

(C) $\tan x$

(D) $\dfrac{(\pi - x)}{(\pi - 2x)}$

4 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

(A) $y = (x-1)^2(x+2)^3$

(B) $y = (x-2)^2(x+1)^3$

(C) $y = (x+1)^2(x-2)^3$

(D) $y = (x+2)^2(x-1)^3$

5 Given $\sin\theta = -\frac{2}{3}$ and $\tan\theta < 0$, then the value of $\cos\theta$ is:?

(A) $\frac{1}{3}$

(B) $\frac{\sqrt{5}}{3}$

(C) $-\frac{\sqrt{5}}{3}$

(D) $-\frac{\sqrt{5}}{2}$

6 Which of the following functions is NOT even?

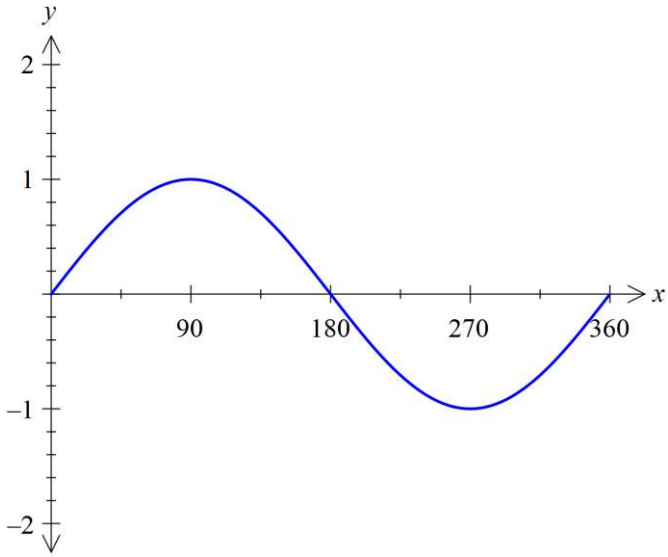
(A) $f(x) = (x-2)^2$

(B) $f(x) = |x|$

(C) $f(x) = \cos x$

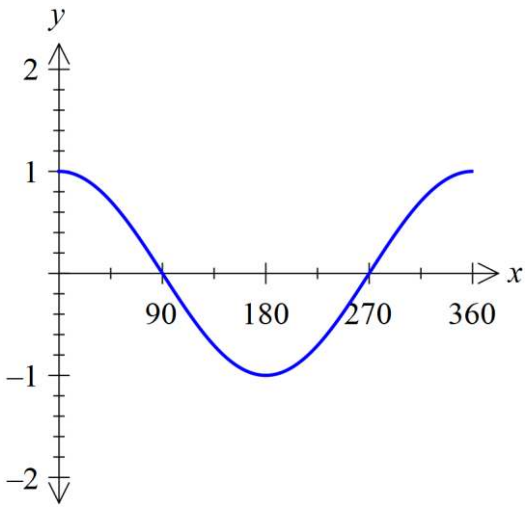
(D) $y = 2^x + 2^{-x}$

7 The graph of $y = \sin x$ is shown below for $0° \le x \le 360°$.

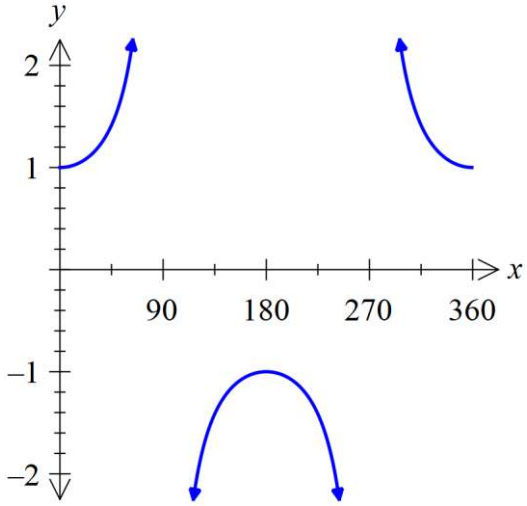


Which of the following is the graph of $y = \text{cosec} x$?

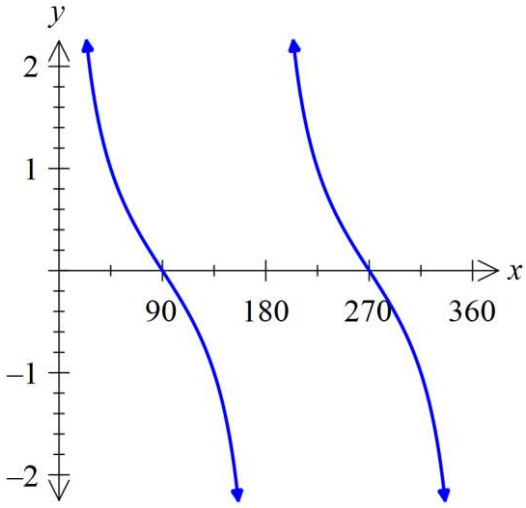
(A)



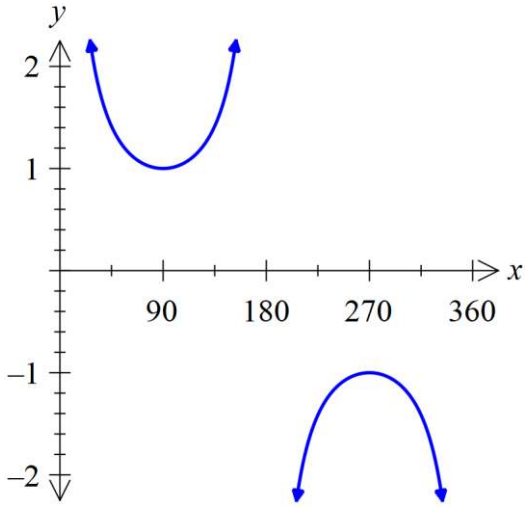
(B)



(C)



(D)



8 The gradient of the normal to the curve $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$ is:

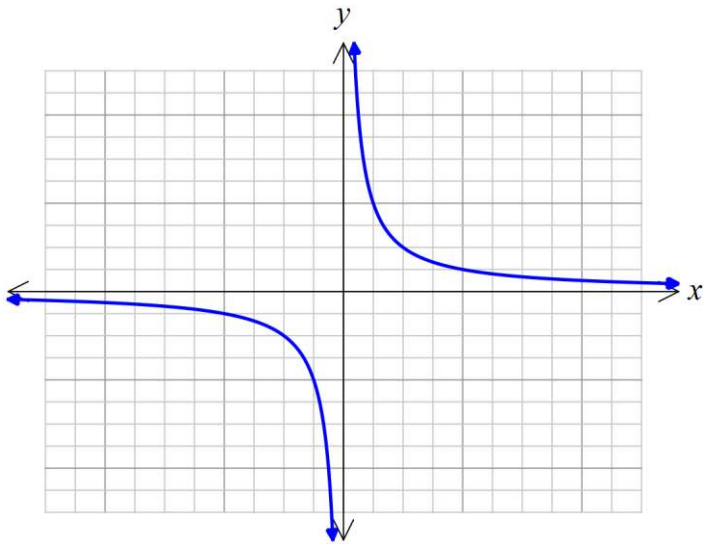
(A) –4

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) 4

9 The graph of $f(x) = \frac{1}{x}$ is shown below.



The domain and range of the function $g(x) = 2f(x) + 1$ is:

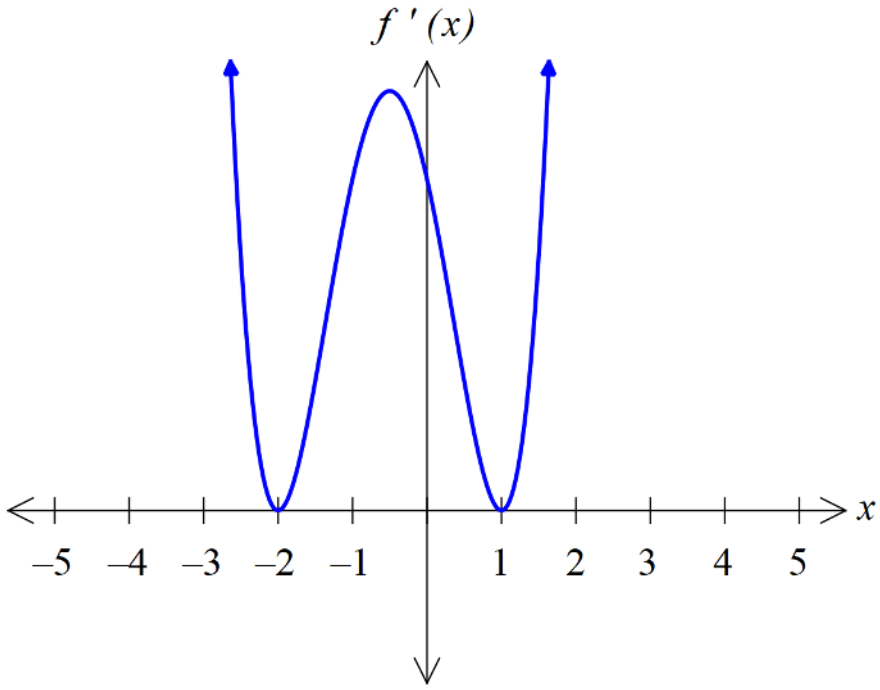
(A) Domain: real x, $x \neq 0$. Range: real y, $y \neq -1$

(B) Domain: real x, $x \neq 2$. Range: real y, $y \neq -1$

(C) Domain: real x, $x \neq 0$. Range: real y, $y \neq 1$

(D) Domain: real x, $x \neq 2$. Range: real y, $y \neq 1$

10 The graph of the derivative of $y = f(x)$ is shown below.



Which of the following is true?

(A) $y = f(x)$ has three stationary points

(B) $y = f(x)$ has four stationary points

(C) $y = f(x)$ is never decreasing

(D) $y = f(x)$ is always increasing

End of Section I

Section II

71 marks
Attempt Questions 11-26
Allow about 1 hour and 45 minutes for this section

Question 11 (2 marks)

Solve $\sin x = -\dfrac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$.

Question 12 (4 marks)

Consider the quadratic function $f(x) = -2x^2 + 12x - 18$.

(a) Express $f(x)$ in the form $a(x-h)^2 + k$ and state the vertex of $y = f(x)$. **2**

(b) Find the value of the discriminant of $f(x)$. **1**

(c) State the number of x-intercepts for the graph $y = f(x)$. **1**

Question 13 (5 marks)

If $a = \log_{10} x$, $b = \log_{10} y$ and $c = \log_{10} z$, express the following in terms of *a, b* and *c*.

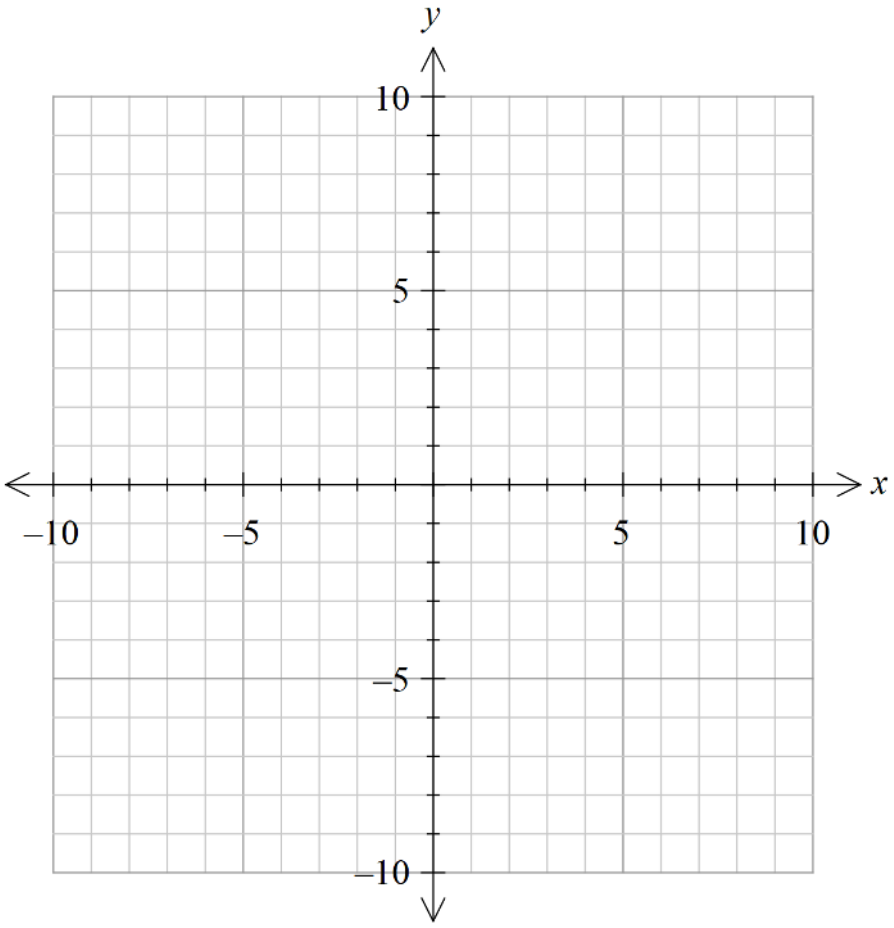
(a) $\log_{10}\left(\dfrac{x^2\sqrt{y}}{z}\right)$ **3**

(b) $\log_{100} y$ **2**

Question 14 (3 marks)

Given that $f(x) = |x|$ and $g(x) = 3 - \frac{x}{2}$.

(a) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same number plane below, labelling the point(s) of intersection. **2**



(b) Hence, or otherwise, solve $|x| < 3 - \frac{x}{2}$. **1**

_____

_____

_____

_____

_____

Question 15 (8 marks)

Differentiate the following functions, expressing your answers in factored form where possible:

(a) $y = \dfrac{3x + x\sqrt{x}}{\sqrt{x}}$ **2**

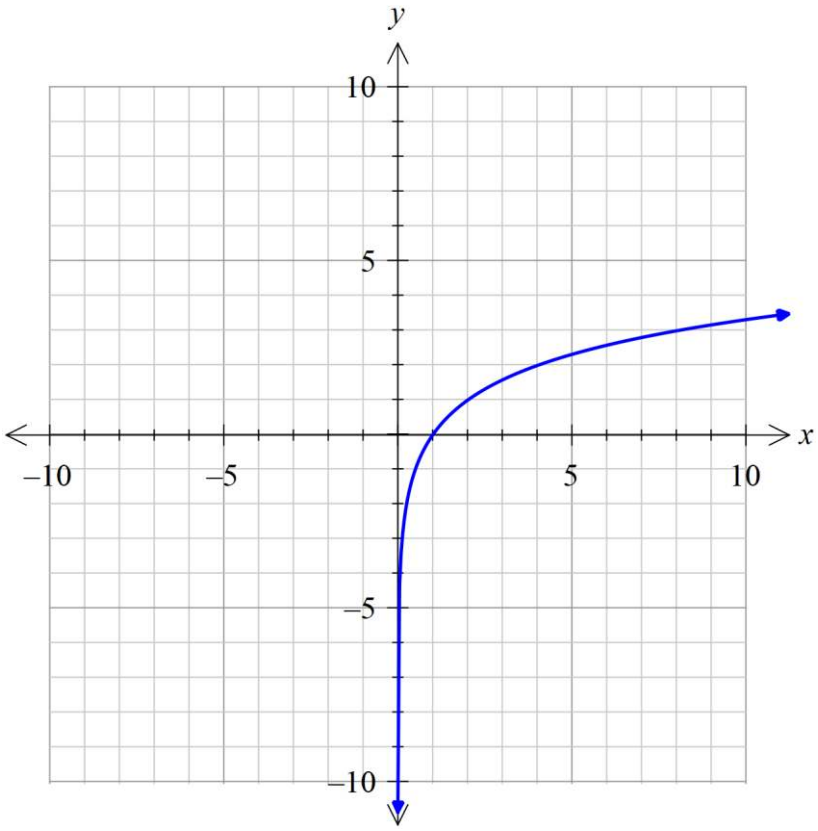
(b) $y = 2x\left(1+3x^2\right)^2$ **2**

(c) $f\left(x\right) = \dfrac{x+3}{2x+1}$ **2**

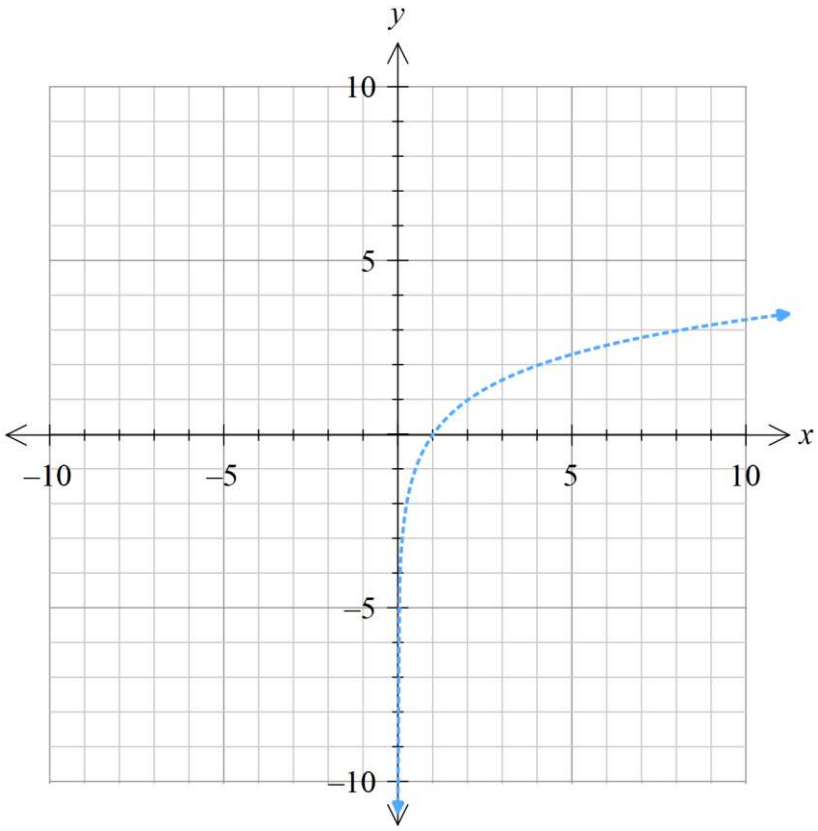
(d) $y = x^2e^x$ **2**

Question 16 (6 marks)

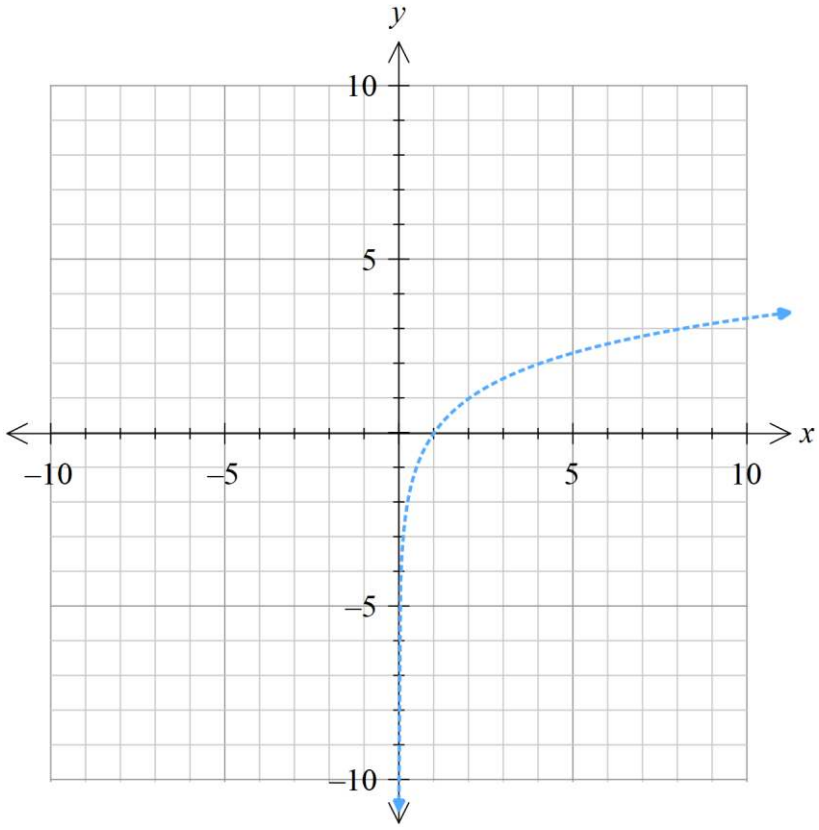
The graph of $y = \log_2 x$ is shown on the number plane below. It is also shown dotted on the following number planes.



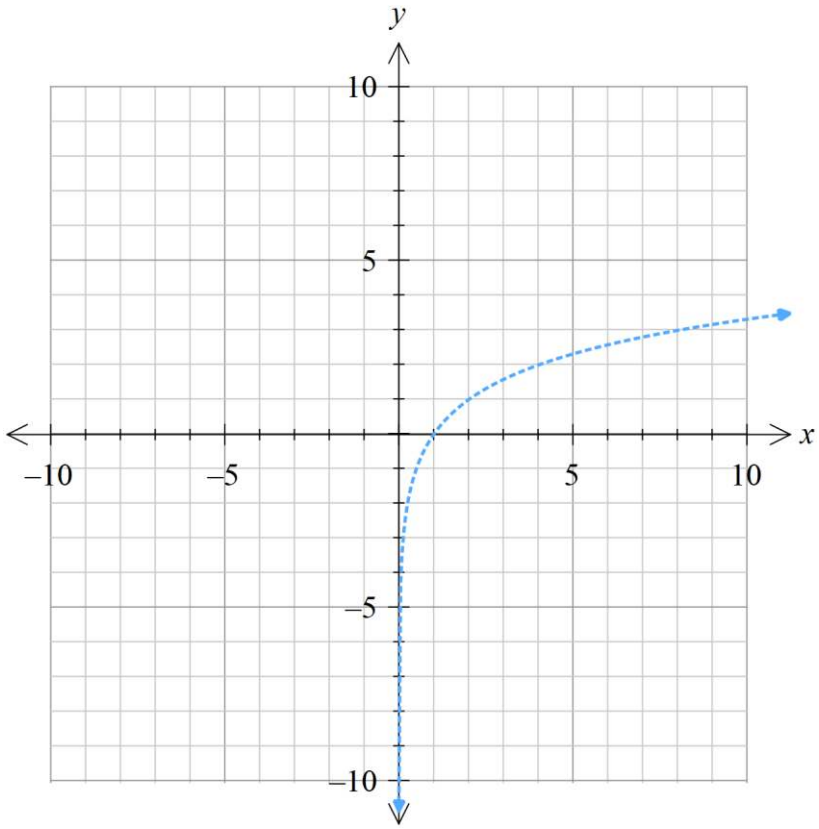
(a) Sketch the graph of $y = \log_2 (x+3)$ on the number plane below, labelling any intercepts and asymptotes. **2**



(b) Sketch the graph of $y = \log_2 x^3$ on the number plane below, labelling any intercepts and asymptotes. **2**



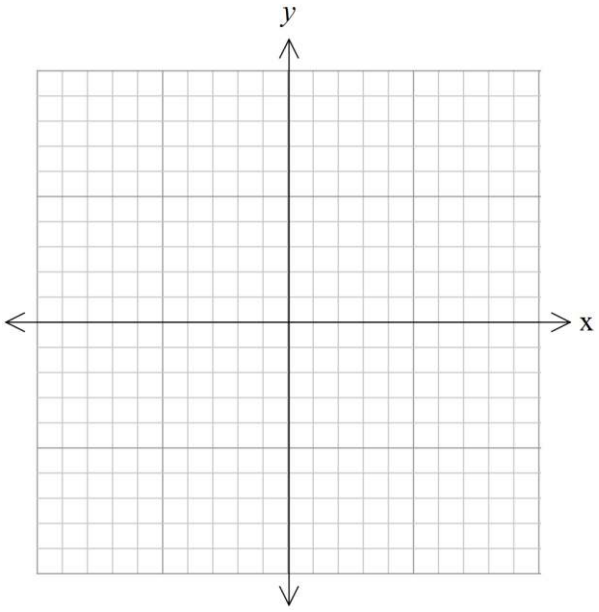
(c) Sketch the graph of $y = 2^x$ on the number plane below, labelling any intercepts and asymptotes. **2**



Question 17 (5 marks)

Consider the semi-circle defined by function $f(x) = \sqrt{8 - x^2}$.

(a) Sketch the graph of $y = f(x)$. **1**

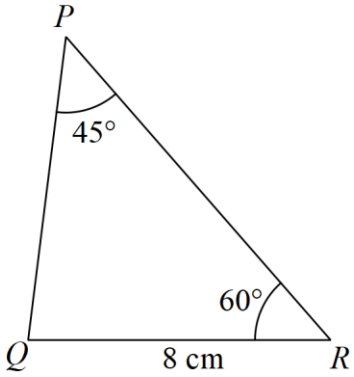


(b) Show that $f'(x) = \dfrac{-x}{\sqrt{8 - x^2}}$. **1**

(c) Find the coordinates of the point(s) on the curve on $y = f(x)$ where the tangent is parallel to $y = x - 1$. **3**

Question 18 (4 marks)

ΔPQR is shown in the diagram below.



NOT TO SCALE

(a) Prove that $PQ = 4\sqrt{6}$ cm. **2**

(b) Find the area of triangle PQR, correct to 3 significant figures. **2**

Question 19 (4 marks)

In a yeast culture, cell numbers are given by $N = 100 \times (2.7)^{1.03t}$, where t is measured in hours after the cells are introduced to the culture.

(a) What is the initial number of cells? **1**

(b) How many cells, correct to the nearest whole number, will be present after 6 hours? **1**

(c) How long will it take for the population to reach one million cells? Give your answer correct to the nearest hour. **2**

Question 20 (2 marks)

Prove the identity $(1-\cos x)(1+\sec x) = \sin x \tan x$.

Question 21 (7 mark)

A particle is travelling in one-dimension. Its displacement in metres at time t seconds is given by $x = t^3 - 6t^2 + 9t + 3$.

(a) Find the equations for velocity and acceleration in terms of *t*. **2**

(b) Find the two times, t_1 and t_2, when the particle is at rest. **2**

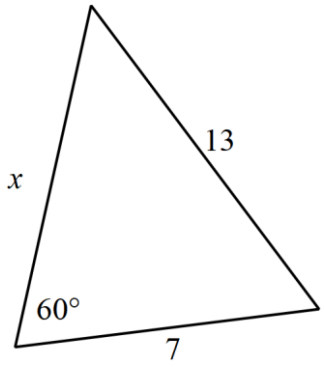
(c) Complete the table of values below. **2**

t	0	t_1 = _____	t_2 = _____
x			

(d) Find the distance travelled by the particle in the first three seconds. **1**

Question 22 (4 marks)

The diagram shows a triangle with sides 7 cm, 13 cm and x cm, an angle of $60°$ as marked.



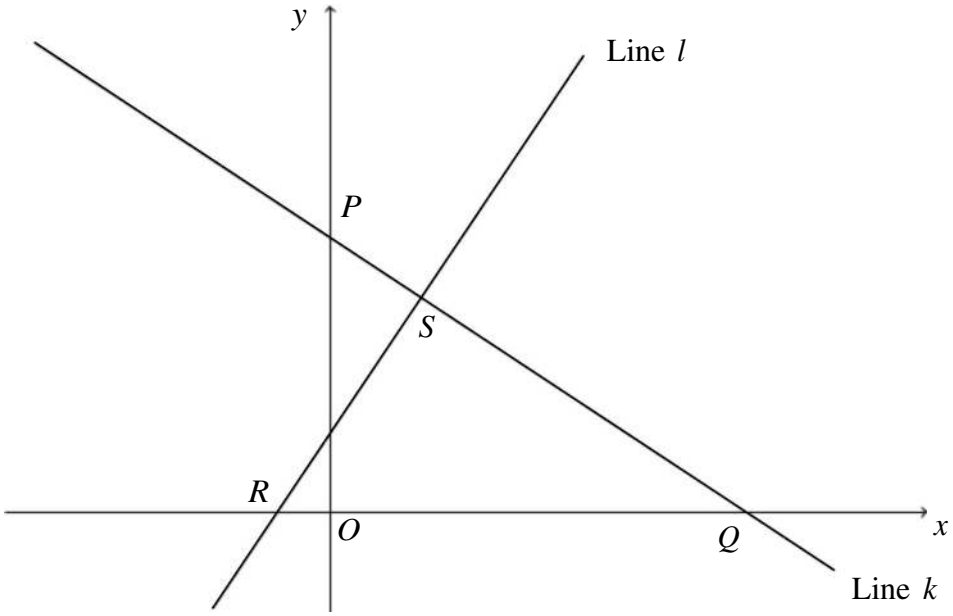
NOT TO SCALE

(a) Use the cosine rule to show that $x^2 - 7x = 120$. **2**

(b) Hence, find the value of x. **2**

Question 23 (7 marks)

In the diagram, the line l has equation $3x-2y+3=0$ and the line k passes through the points $P(0,8)$ and $Q(12,0)$.



NOT TO SCALE

(a) Line l cuts the x-axis at R. Find the coordinates of R. **1**

(b) Show that lines l and k are perpendicular. **2**

(c) Find the equation of the line k and show it is $2x + 3y - 24 = 0$. **1**

(d) Find the coordinates of S, the point of intersection of k and l. **2**

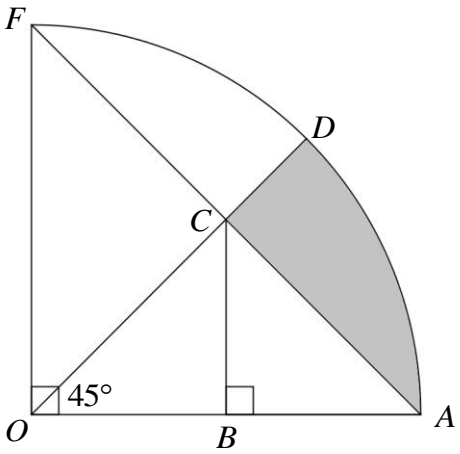
(e) Hence, or otherwise, find the area of ΔQRS. **1**

Question 24 (2 marks)

By taking logarithms of both sides, or otherwise, solve $2 \times 3^{x-2} = 36^{x-1}$, expressing your answer correct to 2 decimal places.

Question 25 (5 marks)

A quarter of a circle, with centre O and radius $OA = OF = OD = 6$ cm is shown below, with CB the perpendicular bisector of OA.



NOT TO SCALE

(a) Find the exact length of arc AF. **1**

(b) Find the exact area of the sector bounded by OA, OD and arc AD. **2**

(c) Find the area of the shaded region ACD, correct to the nearest millimetre. **2**

Question 26 (3 marks)

Solve $3\cos^2\theta + 4\sin\theta\cos\theta = 0$ for $0° \le \theta \le 360°$ **3**

End of Assessment Task

Section II extra writing space

If you use this pace, clearly indicate which questions you are answering