Examination number……

Roseville College
Mathematics Department

2020

YEAR 11 YEARLY
EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours and 30 minutes
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- For Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 80

Section I – 8 marks (pages 2 – 4)

- Attempt Questions 1 – 8
- Allow about 15 minutes for this section

Section II – 72 marks (pages 5 – 24)

- Attempt Questions 9 – 31
- Allow about 2 hours and 15 minutes for this section

Section I

8 marks
Attempt Questions 1–8
Allow about 15 minutes for this section

Use the **multiple-choice answer sheet** for Questions 1 – 8

1. Which of the following is true for the quadratic equation $3x^2 + x + 4 = 0$?

A. The roots are rational

B. The roots are not real

C. The roots are equal

D. The roots are irrational

2. Which set of points below is a function?

A. (–6, 7) , (7 , 6) , (–4, 5) and (–6, –4)

B. (–6, 7) , (7 , –4) , (–6, 5) and (7, –2)

C. (–6, 7) , (7 , 5) , (–4, 5) and (7, –6)

D. (–6, 7) , (7 , –6) , (–4, 5) and (5, –4)

3. The solution to $|x-3|-8=0$ is

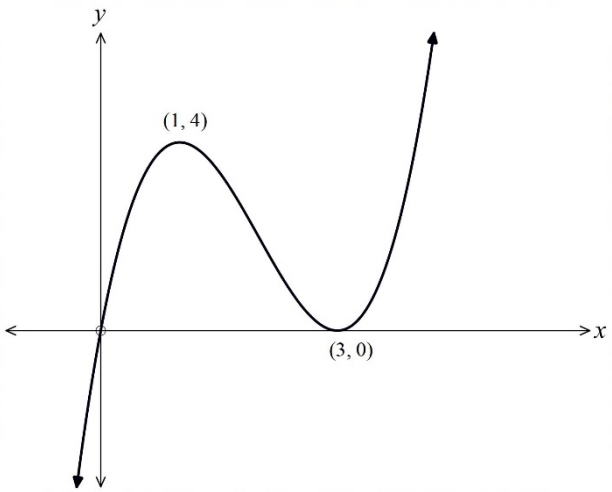
A. x = 11

B. x = 5, -11

C. x = -5, 11

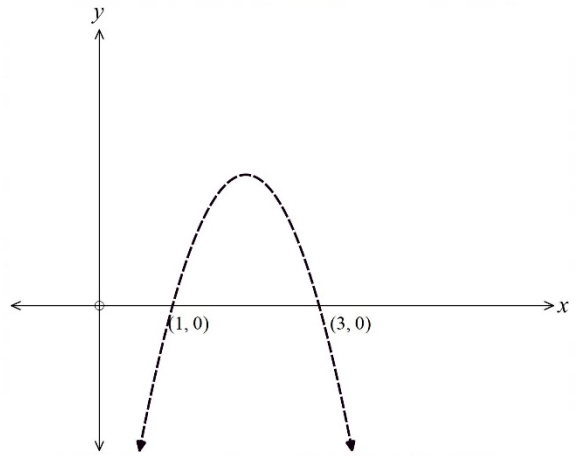
D. x = 5, 11

4. The graph of $y = f(x)$ is shown below.

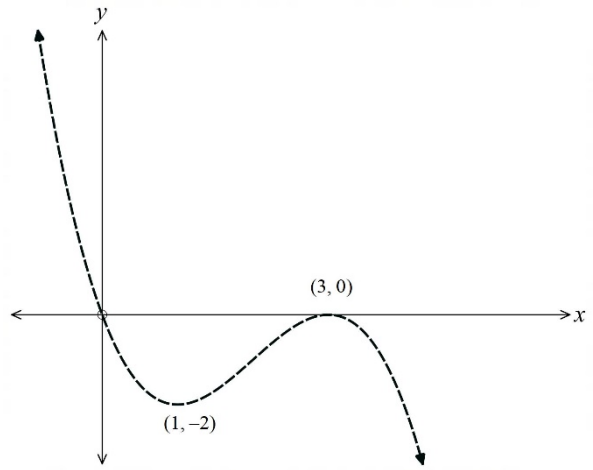


Which graph could represent $y = f'(x)$

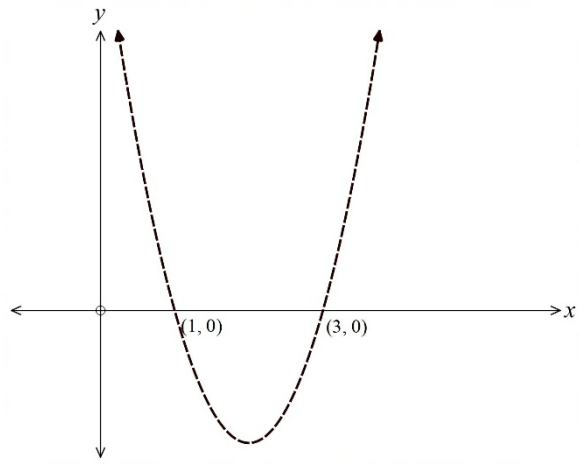
A.



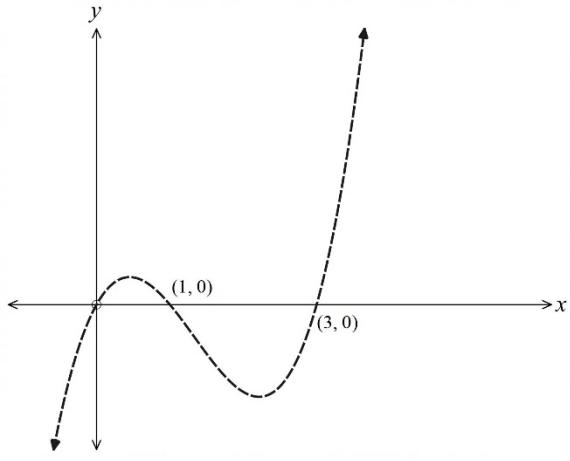
B.



C.



D.



5. Find the exact value of $\tan \frac{5\pi}{3}$.

A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

6. Leo owns five blue and seven red ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother. What is the probability that both of the ties, are the same colour?

A. $\frac{1}{2}$

B. $\frac{5}{33}$

C. $\frac{31}{66}$

D. $\frac{31}{72}$

7. If $a = \log_b(x)$, which equation is true?

A. $a = b^x$

B. $b = x^a$

C. $x = b^a$

D. $x = a^b$

8. The population (P) of a town is given by $P = 2\,000\,000\,e^{0.03t}$ where t is time in years. What is the population in 19 years?

A. 3 536 534

B. 2 000 123

C. 597 734 801

D. 300 090

Roseville College
Mathematics Department

2020 YEAR 11 FINAL EXAMINATION

Mathematics Advanced

Student Number

Section II Answer Booklet

72 marks
Attempt Questions 9 – 31
Allow about 2 hours and 15 minutes for this section

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet.
  If you use this space, clearly indicate which question you are answering.

Question 9 (2 marks)

Find the equation of the axis of symmetry of the parabola $y = 5 + 4x - x^2$ and the maximum value of the expression $5 + 4x - x^2$ **2**

……

……

……

……

……

……

……

……

……

……

……

Question 10 (3 marks)

Find the value(s) of k for which the equation $x^2 - (k+3)x + 4k = 0$ has exactly one root. **3**

……

……

……

……

……

……

……

……

……

……

……

……

Question 11 (3marks)

Find the simultaneous solution for x and y if $2x - 3y = 2$ and $5x + 7y = 34$. **3**

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

Question 12 (4 marks)

(a) Simplify the expression $\frac{a^2-3a}{2ax} \times \frac{4a^2x}{2ax-6x}$, giving your answer as a simple fraction. **2**

……

……

……

……

……

……

……

……

……

(b) Solve the equation $\frac{6x+10}{3} = x + 2$. **2**

……

……

……

……

……

……

……

……

……

……

Question 13 (3 marks)

If $f(x) = x^2$ and $g(x) = 2x + 1$

(a) Find the composite function $f\left(g(x)\right)$ **1**

…..……

……

……

……..…….

……..

……..…….

……..

(b) What is the domain and range of the composite function? **2**

……

……

…….

……

……

……

……

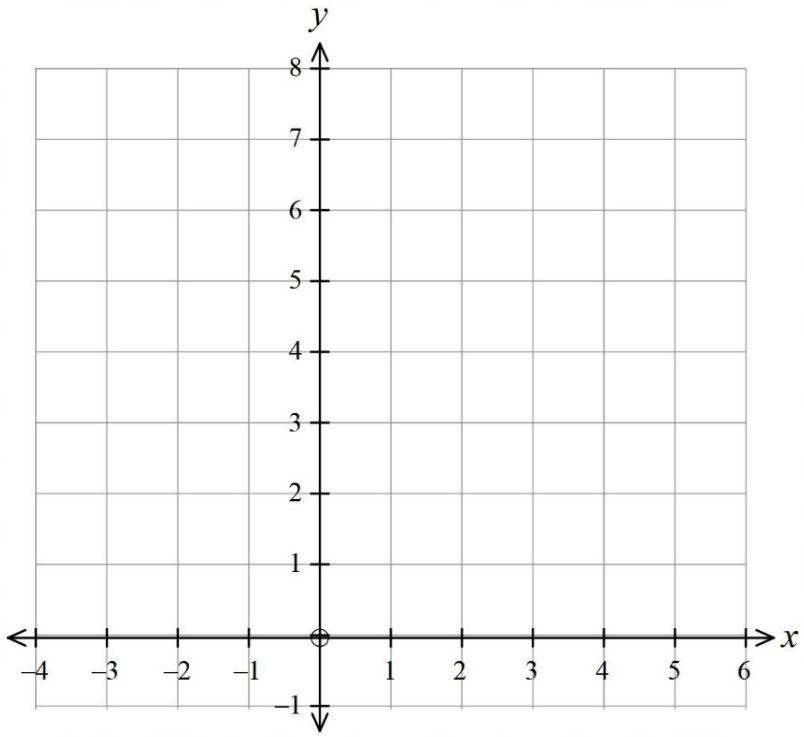
……

……

……

Question 14 (3 marks)

(a) On the axes below, draw the graphs of $y = |x - 3|$ and $y = |3x + 3|$. **2**



……

……

……

……

……

……

(b) Use the graph to find solutions to $|x - 3| = |3x + 3|$. **1**

……

……

……

……

Question 15 (3 marks)

(a) Find the equation of the line which passes through the points (–1, 14) and (3, 6). **2**

……

……

……

……

……

……

……

……

……

……

……

(b) Show that the points (–1, 14), (3, 6) and (6, 0) lie on the same line. **1**

……

……

……

……

……

……

……

……

……

……

Question 16(3 marks)

(a) Find the centre and radius of the circle represented by the equation: **2**

$x^2 - 4x + y^2 + 6y + 4 = 0$.

……

……

……

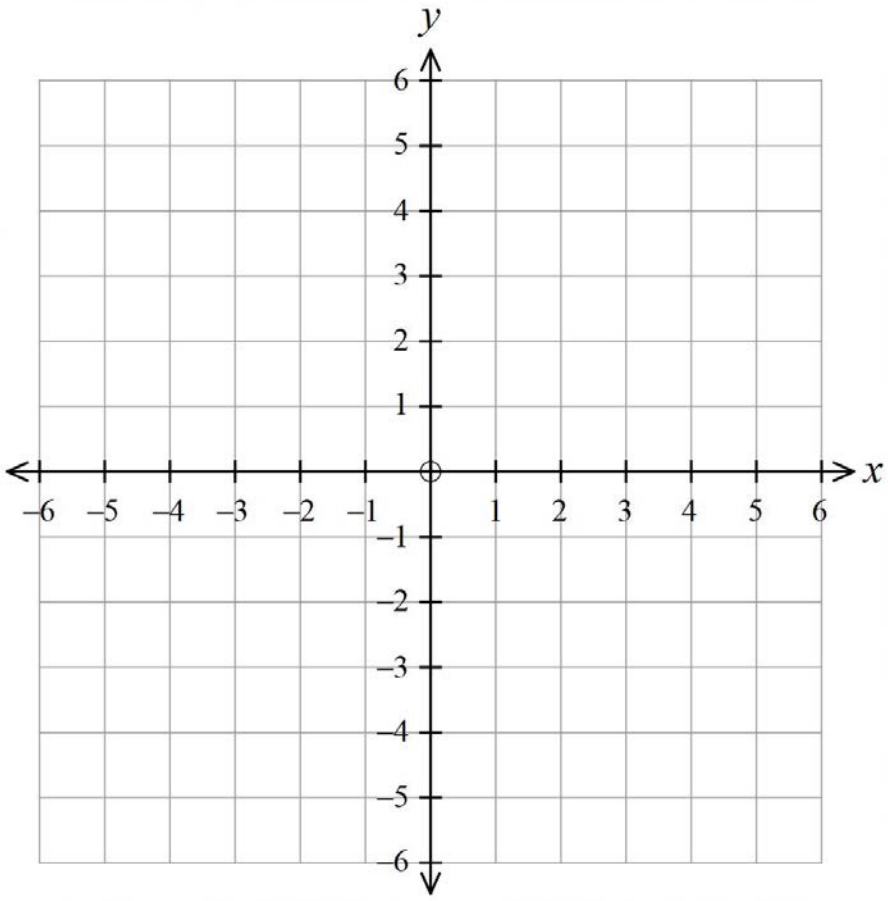
……

……

……

……

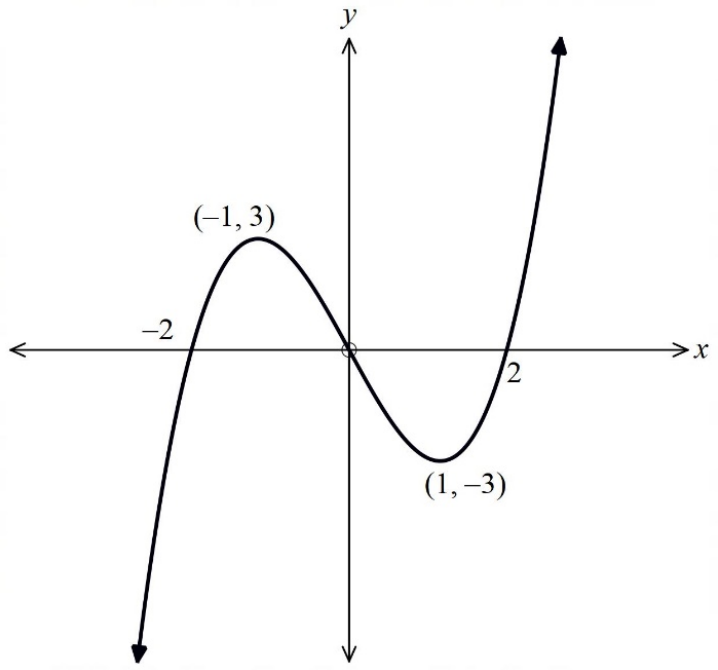
(b) Draw the circle on the axes below. **1**



Question 17 (3 marks)

(a) Classify the function whose graph is shown below as odd, even or neither. **1**

Support your choice with reasons.



……

……

……

……

(b) Classify the function whose equation is $y = x^4 + 4x^2 - 5$ as odd, even or neither. **2**

Support your choice with calculations.

……

……

……

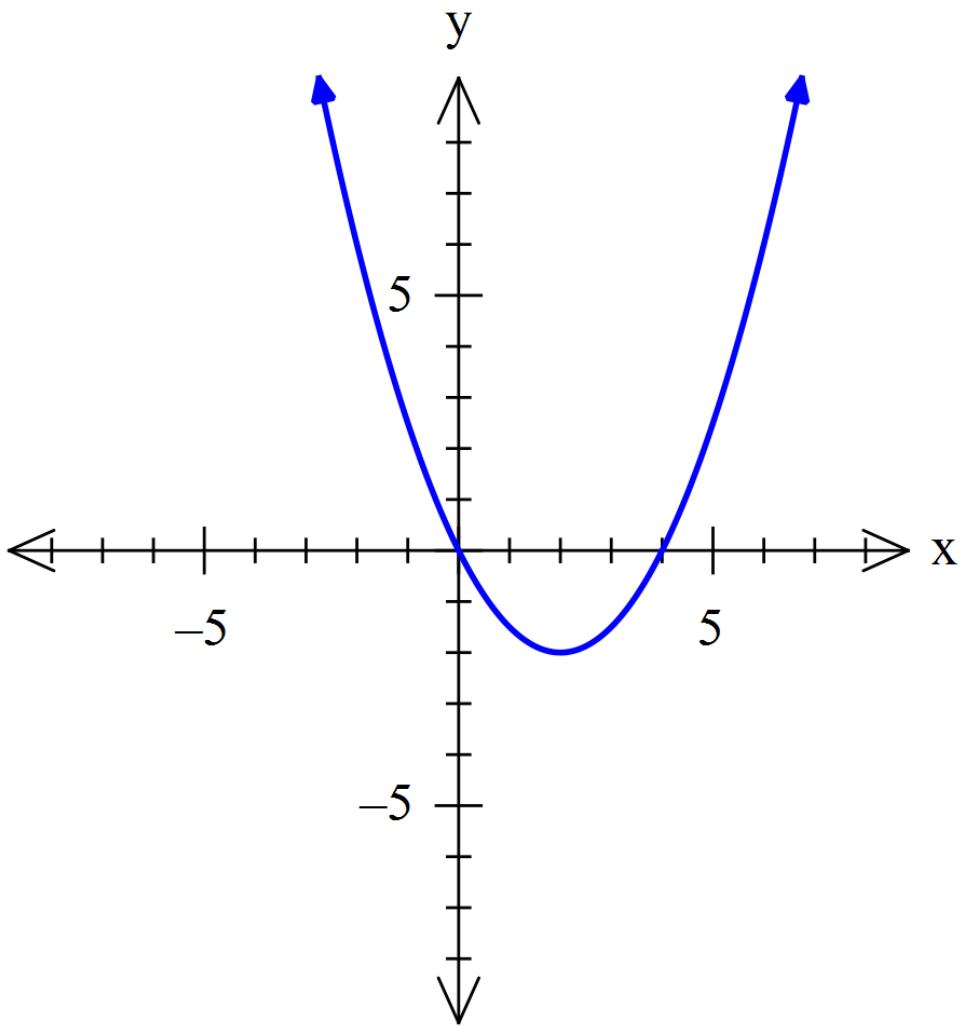
……

……

……

Question 18 (2 marks)

The sketch below shows the function $y = f(x)$. On the same number plane draw $y = -f(x)$ **2**



Question 19 (3 marks) **3**

Show that $\sin\alpha \cot\alpha - \cos\alpha \sin^2\alpha = \cos^3\alpha$.

……

……

……

……

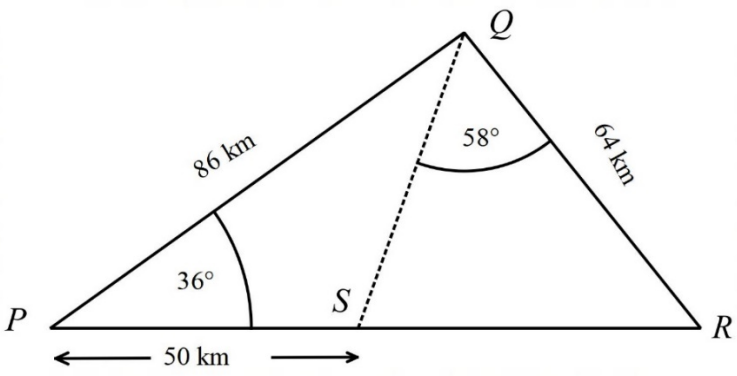
……

……

……

Question 20 (5 marks)

Triangles PQS and QRS have a common side QS, as shown.



(a) Calculate the length of QS, correct to 3 significant figures **2**

……

……

……

……

……

……

……

……

(b) Find the area of ΔPQR, correct to the nearest square kilometre. **3**

……

……

……

……

……

……

……

……

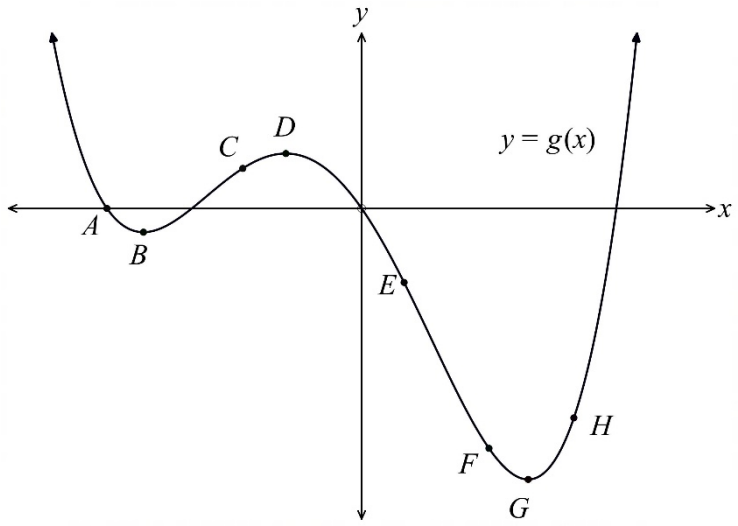
……

……

Question 21 (3 marks)

The curve $y = g(x)$ is shown below.

Eight points on the curve are labelled A – H.



(a) Name the point(s) where the tangent to the curve is horizontal. **1**

……

……

(b) Name the point(s) where the curve is increasing at a decreasing rate. **1**

……

……

(c) Name the point where the curve is changing concavity. **1**

……

……

Question 22 (3 marks)

Gainsborough High is a new school and they decide to choose two colours to be used on the school sports uniform and crest.

There are several suggestions for the primary and secondary colours, and these are shown on the, partially completed, array below.

They decide to choose a pair of colours at random.

Secondary ↓ / Primary →	Crimson	Maroon	Navy Blue	Green	Black
Orange	C O	M O	N O	G O	
White	C W	M W	N W		
Sky Blue	C S	M S			
Yellow	C Y				

(a) Complete the cells in the array. **1**

(b) If there are no restrictions on the colour combinations, what is the probability that the sports colours include at least one shade of Blue? **1**

…….

…….

…….

…….

(c) Given that the primary colour chosen is not Green, what is the probability that the sports colours include Orange and/or Black? **1**

…….

…….

…….

…….

…….

Question 23 (4 marks)

(a) Given that $y = 3x^4(2-x)^5$, find $\frac{dy}{dx}$. It is not necessary to simplify your answer. **2**

.

……

……

……

……

……

……

……

……

……

……

(b) Find $\frac{d}{dx}\left(\frac{\sqrt{x^3}}{2x^3+5x}\right)$. **2**

There is no need to simplify your answer.

……

……

……

……

……

……

……

……

……

……

Question 24 (3 marks) **3**

Use the result $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ to show that if $f(x) = 2x^2$, then $f'(x) = 4x$.

……

……

……

……

……

……

……

……

……

……

……

……

Question 25 (3 marks) **3**

A tangent is drawn to the curve $y = \frac{x^3-x^2-9x}{2}$ at the point where $x = \sqrt{3}$.

Find the angle of inclination of the tangent to the positive *x*-axis.

……

……

……

……

……

……

……

……

……

……

……

Question 26 (4 marks)

A large tank of liquid which contains L litres of a toxic chemical is being drained.
The amount of chemical in the tank at time t minutes, is given by:

$$L = 120(40 - t)^2$$

(a) At what rate is the water draining out of the tank after 6 minutes? **2**

……

……

……

……

……

……

(b) How long will it take for the tank to be completely empty? **2**

……

……

……

……

……

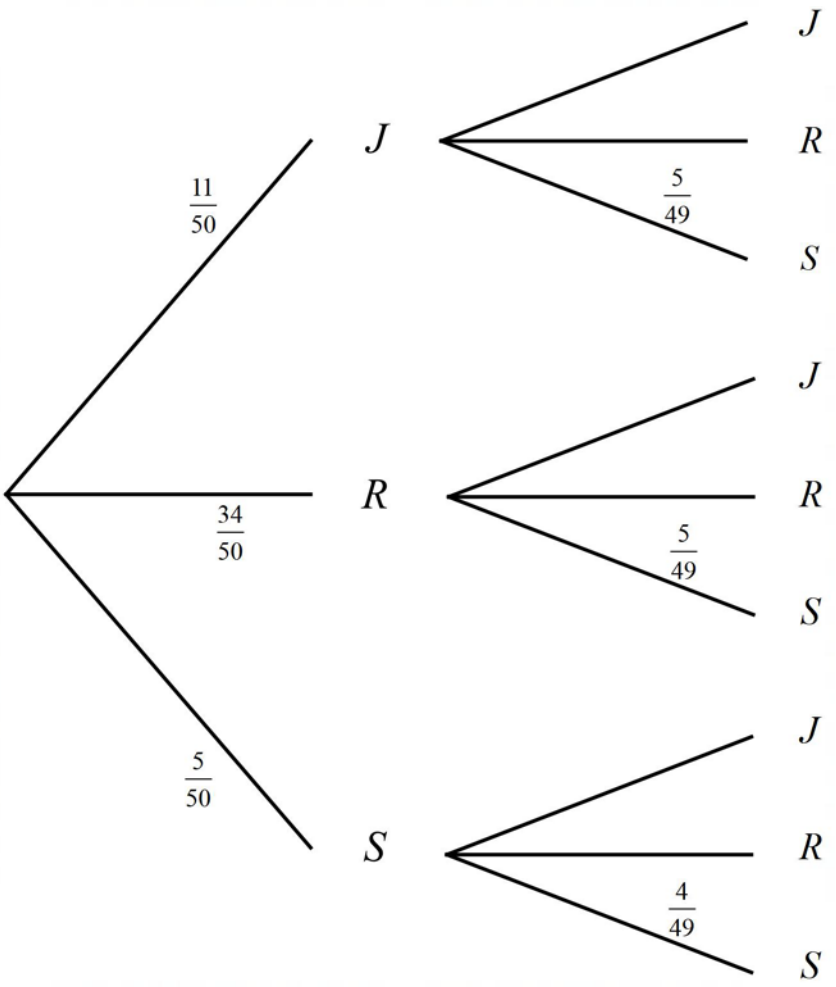
……

……

Question 27 (4 marks) **4**

Employees in a company are classified as Junior if aged less than 18, Senior if aged over 60 or Regular for all others. There are 50 employees in the company and 2 are to be chosen to attend a conference.

The probability tree diagram has been started to show how the two employees could be chosen.



Complete the missing probabilities on the probability tree and hence find the probability that the two employees chosen will come from different categories.

……

……

……

……

……

……

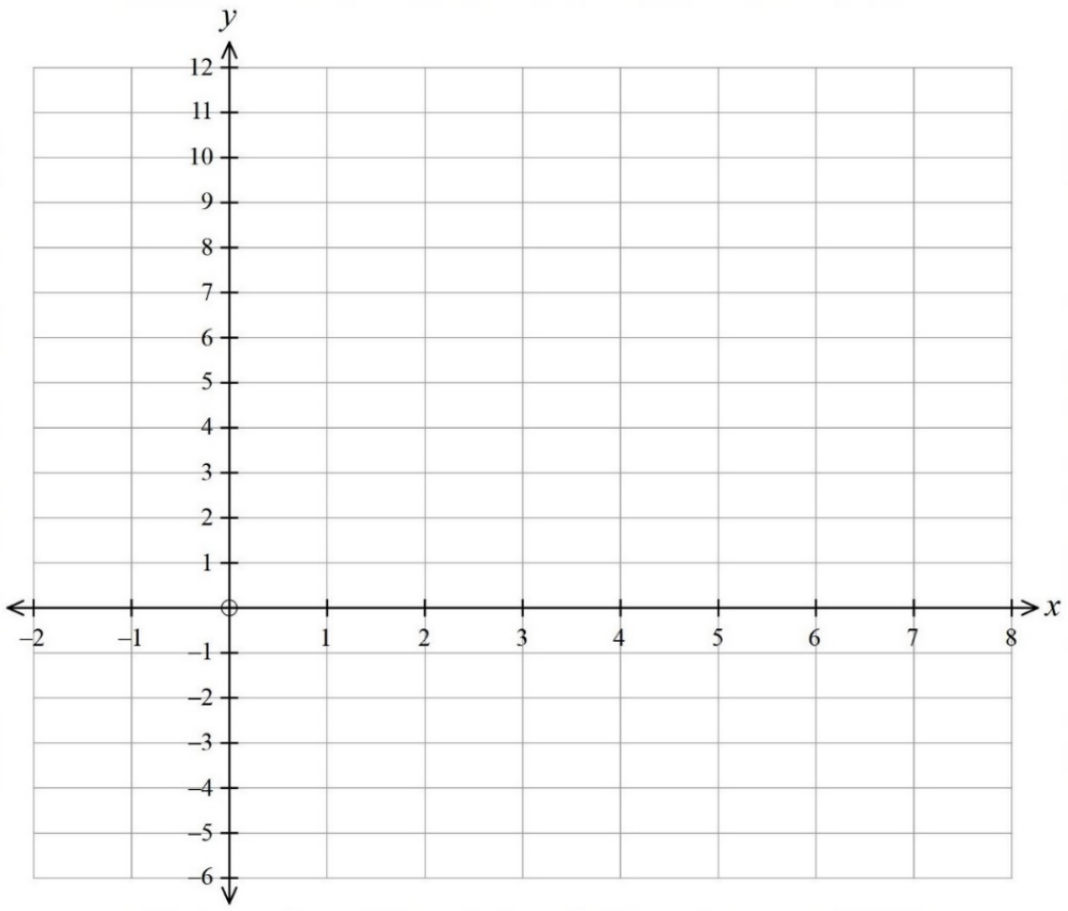
……

……

Question 28 (3 marks)

By completing the table of values, or otherwise, plot the graphs of $y = \log_2 x$ and $y = 3\log_2 x + 1$ on the axes below. **3**

x	$\frac{1}{2}$	1	2	4	8
$y = \log_2 x$					
$y = 3\log_2 x + 1$					



Question 29 (2 marks)

2

Given that $f(x) = 5e^{6x}$ find the exact value of $f'(1)$.

……

……

……

……

……

……

……

……

……

……

Question 30 (3 marks)

(a) Evaluate $\log_a\left(\sqrt{a}\right)^5$. **1**

……

……

……

……

(b) Solve $\log_2(x+2) - \log_2(x-5) = 3$. **2**

……

……

……

……

……

……

……

Question 31 (3 marks)

(a) Given that $y = 2^x$, show that $x = \frac{\log_e y}{\log_e 2}$. **1**

……

……

……

……

……

……

……

(b) Hence or otherwise show that y can be written as $y = e^{x(\log_e 2)}$. **1**

……

……

……

……

……

……

……

(c) Show that $\frac{dy}{dx} = (\log_e 2)y$. **1**

……

……

……

……

……

……

……

End of Paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

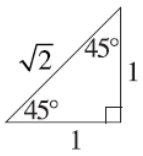
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

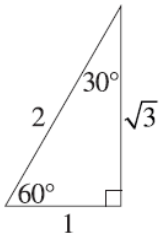
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

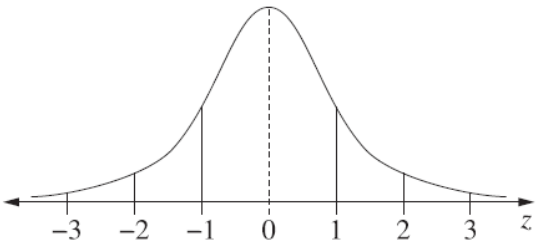
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Roseville College
Mathematics Department

2020 Year 11 Final Examination
Mathematics Advanced

Name _____ Teacher _____

Section I – Multiple Choice Answer Sheet

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ ← correct C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○